Total No. of Questions : 12] [Total No. of Printed Pages : 6

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F. E. Examination - 2009

ENGINEERING MATHEMATICS - II

(2003 Course)

Time : 3 Hours] [Max. Marks : 100

Instructions :

(1) In section I, attempt Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4, Q. No. 5 or Q. No. 6.

(2) In section II, attempt Q. No. 7 or Q. No. 8, Q. No 9 or Q. No. 10, Q. No. 11 or Q. No. 12.

(3) Answers to the ***two sections*** *should be written in* ***separate answer-books****.*

(4) Figures to the right indicate full marks.

(5) Neat diagrams must be drawn wherever necessary.

(6) Use of non-programmable electronic pocket calculator is allowed.

(7) Assume suitable data, if necessary.

SECTION - I

Q.1) (A) Form the differential equation of family of circles of fixed radius a and centre on positive side of y-axis. **[05]**

(B) Solve **any three** of the following Differential Equations : **[12]**

(1) $\left[1 + e^{x/y}\right] dx + e^{x/y} . \left[1 - \frac{x}{y}\right] dy = 0$

(2) $\sin y \frac{dy}{dx} = (1 - x\cos y) \cos y$

(3) $(3y - 7x + 7)\, dx + (7y - 3x + 3)\, dy = 0$

(4) $\left[xy^2 - e^{1/x^3}\right] dx - x^2y\, dy = 0$

(5) $\frac{dy}{dx} = (x + y + 1)$

OR

Q.2) (A) Form the differential equation whose general solution is :

$y = A \cdot e^{-2x} + B \cdot e^{-3x}$ **[05]**

(B) Attempt **any three** of the following Differential Equations : **[12]**

(1) $\frac{dy}{dx} = 1 - x \tan(x - y)$

(2) $(y^3 - 2x^2y)\,dx + (2xy^2 - x^3)\,dy = 0$

(3) $(y^4 + 2y)\,dx + (xy^3 + 2y^4 - 4x)\,dy = 0$

(4) $(x^2y^2 + 5xy + 2)\,y \cdot dx + (x^2y^2 + 4xy + 2)\,x \cdot dy = 0$

(5) $\cos x \frac{dy}{dx} + y \sin x = \sec x$

Q.3) Solve **any three** of the following :

(a) The distance 'x' descended by a person falling freely under gravity by means of a parachute, satisfy the differential equation

$\left(\frac{dx}{dt}\right)^2 = k^2\left[1 - e^{\frac{-2gx}{k^2}}\right]$ where g and k are constants. If he falls

from rest, show that $x = \frac{k^2}{g} \log \cosh\left(\frac{gt}{k}\right)$ **[06]**

(b) The series electrical circuit consisting of inductance 'L' and resistance 'R' is connected to e.m.f. $E_o \cdot e^{-at}$, where E_o and a are constants. Show that the current as a function of time is given by

$$i = \frac{E_o}{R - aL}\left[e^{-at} - e^{-\frac{R}{L}t}\right]$$ **[05]**

(c) The steady heat flow through the spherical shell of radius $r\ (r_1 \le r \le r_2)$ satisfy the differential equation

$r \cdot \frac{d^2u}{dr^2} + 2\frac{du}{dr} = 0$, if temperature $u = u_1$ when $r = r_1$ and $u = u_2$ when $r = r_2$. Find the temperature u interms of r. **[06]**

(d) A body cools from 100°C to 70°C in 15 minutes when surrounding temperature is 30°C. Find time when the temperature of the body will be 40°C **[05]**

OR

Q.4) Attempt **any three** of the following :

(a) Find the Orthogonal Trajectories of $x^2 + 2y^2 = c^2$. **[05]**

(b) Find the current 'i' in a circuit consisting of resistance 50 ohms and condenser of capacity 0.02 Farad in a series with e.m.f. 10sin(2t). **[05]**

(c) A steam pipe 20cm in diameter is protected with a covering 6cm thick for which k = 0.0003, in steady state. Find the heat loss per hour through a meter length of the pipe, if the surface of the pipe is at 200°C and the outer surface of covering is at 30°C. **[06]**

(d) An elastic string of natural length 'l' is fixed at a point A. To the lower end of it, a particle of mass 'm' is attached so that the spring is streched to the length '$2l$'. If the particle is dropped from A, show that it descends a distance $l\left(2 + \sqrt{3}\right)$ before coming to rest. **[06]**

Q.5) (A) Find the equation of sphere that touches the given sphere $x^2 + y^2 + z^2 - x + 2y + 2z - 3 = 0$ at the point (1, 1, –1) and passing through the point (0, 0, 3). **[06]**

(B) Find the equation of right circular cylinder of radius '3' with axis along the line
$x + z + 2 = 0 = x - 2y + 4$ **[05]**

(C) Find the equation of cone generated by rotating the line $2x + 3y = 6, z = 0$, about y-axis. **[05]**

OR

Q.6) (A) Show that the plane $4x - 3y + 6z = 35$, is tangential to the sphere $x^2 + y^2 + z^2 - y - 2z - 14 = 0$; and find the point of contact. **[06]**

(B) Find the equation of right circular cone with vertex at origin and axis as the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and semi-vertical anlge 30°. **[05]**

(C) Find the equation of cylinder whose generator is parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and guiding curve is $x^2 + 2y^2 = 1, z = 3$. **[05]**

SECTION - II

Q.7) (A) Obtain Fourier Series Expansion for the function

$f(x) = x - x^2, \quad -1 \leq x \leq 1$ **[08]**

(B) Establish the Reduction formula connecting

I_n to I_{n-2}, where $I_n = \int_0^{\pi/2} x \cdot \sin^n x dx$ **[05]**

(C) Evaluate $\int_2^5 \sqrt{(x-2)^7 (5-x)^9} \cdot dx$ **[04]**

Q.8) (A) The following table gives the vibration of periodic current over a period.

T :	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{3}$	T
A :	1.98	1.30	1.05	1.30	–0.88	–0.25	1.98

Show by periodic harmonic analysis, that there is direct current part of 0.75 amp. in variable current and obtain the amplitude of first harmonic. **[08]**

(B) If $I_n = \int_0^{\pi/2} \cos^n x \cos(nx)\, dx$, prove that

$$I_n = \frac{1}{2} \cdot I_{n-1} = \frac{\pi}{2^{n+1}}$$ **[05]**

(C) Show that $\beta(m, n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1 + x)^{m+n}}\, dx$ **[04]**

Q.9) (A) Trace **any two** of the following curves : **[08]**

(1) $y^2 (x^2 + 4) = x^2 + 2x$

(2) $r = 1 + 2\cos\theta$

(3) $x = a(\theta - \sin\theta)$

$y = a(1 - \cos\theta)$

(B) Show that

$\mathrm{Erf}_c(-x) = 2 - \mathrm{Erf}_c(x)$ **[04]**

(C) Evaluate : $\int_0^a \frac{\log(1 + ax)}{1 + x^2}\, dx$ **[05]**

OR

Q.10) (A) Trace the curve : **(Any Two)** **[08]**

(1) $xy^2 = a(x^2 - a^2),\ a > 0$

(2) $x^{2/3} + y^{2/3} = a^{2/3}$

(3) $r = a\cos(3\theta)$

(B) If $\alpha(x) = \sqrt{\frac{2}{\pi}} \int_0^x e^{-\left(\frac{t^2}{2}\right)} . dt$, show that

$\mathrm{Erf}(x) = \alpha\left[x\sqrt{2}\right]$ **[04]**

(C) Find the length of one loop of the curve $r^2 = a^2 \cdot \cos(2\theta)$. **[05]**

Q.11) (A) Evaluate $\iint_R xy(x+y)\,dx\,dy$, where 'R' is the region bounded by $y = x^2$ and $y^2 = -x$. [05]

(B) Evaluate $\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} \frac{dx\,dy\,dz}{\left(1 + x^2 + y^2 + z^2\right)^2}$ [06]

(C) Find the area inside the cardioid $r = 2a(1 + \cos\theta)$ outside the curve $r = \frac{2a}{(1 + \cos\theta)}$. [05]

OR

Q.12) (A) Find Mean Value (M.V) and Root Mean Square (R.M.S.) value of the ordinate of the cycloid
$x = a(\theta + \sin\theta)$, over the range
$y = a(1 - \cos\theta)$, $\theta = -\pi$ to $\theta = \pi$ [05]

(B) Find the volume of the region enclosed by the cone $z = \sqrt{x^2 + y^2}$ and paraboloid $z = x^2 + y^2$. [06]

(C) Find the centre of gravity of one loop of the curve $r = a\sin(2\theta)$. [05]
