Total No. of Questions : 12] [Total No. of Printed Pages : 6

[3661]-106

F. E. (Semester - II) Examination - 2009

ENGINEERING MATHEMATICS - II

(June 2008 Pattern)

Time : 3 Hours] [Max. Marks : 100

Instructions :

(1) In section I, attempt Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4, Q. No. 5 or Q. No. 6. In section II, attempt Q. No. 7 or Q. No. 8, Q. No. 9 or Q. No. 10, Q. No. 11 or Q. No. 12.

(2) Answers to the ***two*** *sections should be written in* ***separate answer-books.***

(3) Figures to the right indicate full marks.

(4) Neat diagrams must be drawn wherever necessary.

(5) Use of non-programmable electronic pocket calculator is allowed.

(6) Assume suitable data, if necessary.

SECTION - I

Q.1) (A) Form a differential equation whose general solution is $y = Ax + B/x$. **[05]**

(B) Solve the following : **(Any Three)** **[12]**

(a) $(2x + e^x \log y)y\, dx + e^x\, dy = 0$

(b) $(y^4 - 2x^3y)\, dx + (x^4 - 2xy^3)\, dy = 0$

(c) $\dfrac{dy}{dx} = \dfrac{x - y + 3}{2x - 2y + 5}$

(d) $(x^3y^3 + xy)\dfrac{dy}{dx} = 1$

OR

Q.2) (A) Form a differential equation whose general solution is $y = (c_1 + c_2 t)\, e^t$ [05]

(B) Solve the following : **(Any Three)** [12]

(a) $x^4 \frac{dy}{dx} - x^3 y - \sec(xy) = 0$

(b) $(y^4 + 2y)\, dx + (xy^3 + 2y^4 - 4x)\, dy = 0$

(c) $x \frac{dy}{dx} + \frac{y^2}{x} = y$

(d) $x\cos x \frac{dy}{dx} + y\,(x\sin x + \cos x) = 1$

Q.3) Solve **any three** :

(a) A body starts moving from rest, is opposed by a force per unit mass of value cx and resistance per unit mass of value bv^2, where x and v are the displacement and velocity of the particle at that instant. Show that the velocity of the particle is given by $v^2 = \frac{c}{2b^2}\left(1 - e^{-2bx}\right) - \frac{cx}{b}$. [06]

(b) A pipe 20 cms. in diameter contains steam at 150°C and is protected with a covering 5 cms. thick, for which k = 0.0025. If the temperature of the outer surface of the covering is 40°C, find the temperature half way through the covering, under steady-state conditions. [06]

(c) A constant emf 'E' volts is applied to a circuit containing constant resistance 'R' ohms in series with constant inductance 'L' henries. If the initial current is zero, show that the current builds upto half of its theoretical maximum in (Llog2)/R seconds. [05]

(d) If 30% of a radioactive substance disappeared in 10 days, how long will it take for 90% of it to disappear ? [05]

OR

Q.4) Solve **any three** :

(a) A body at temperature 100°C is placed in a room whose temperature is 25°C, and cools to 80°C in 10 minutes. Find the time when the temperature will be 60°C. **[05]**

(b) The distance 'x' descended by a parachuter satisfies the equation $v\frac{dv}{dx} = g\left(1 - \frac{v^2}{k^2}\right)$, where v is velocity, k, g are constants. If v = 0 and x = 0 at time t = 0, show that

$$x = \frac{k^2}{g} \log \cosh \left(\frac{gt}{k}\right).$$ **[06]**

(c) A circuit consists of resistance 'R' ohms and a condenser 'C' farads, connected to a constant e.m.f. If q/C is voltage of the condenser at time t after closing the circuit, show that the voltage at time t is $E\left(1 - e^{-t/RC}\right)$. **[05]**

(d) The amount x of a substance present in a certain chemical reaction at time t is given by $\frac{dx}{dt} + \frac{x}{10} = 2 - 1.5\ e^{-t/10}$. If at t = 0, x = 0.5, find x at t = 10. **[06]**

Q.5) (A) Find Fourier series to represent the function $f(x) = \pi^2 - x^2$ in the interval $-\pi \le x \le \pi$ and $f(x + 2\pi) = f(x)$. Deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots = \frac{\pi^2}{8}.$$ **[07]**

(B) If $I_n = \int_0^{\pi/2} x\cos^n x\ dx$, prove that

$$I_n = -\frac{1}{n^2} + \frac{n-1}{n} I_{n-2}$$ **[05]**

(C) Evaluate $\int_3^7 (x - 3)^{1/4}\ (7 - x)^{1/4}\ dx$ **[04]**

OR

Q.6) (A) Evaluate $\int_0^\infty x^9 e^{-2x^2} dx$ [04]

(B) Find reduction formula for

$$I_n = \int_0^{\pi/4} \sec^n x \, dx.$$ [05]

(C) The following table gives variation of periodic current over a period :

t sec.	0	T/6	T/3	T/2	2T/3	5T/6	T
A (amp)	1.98	1.30	1.05	1.30	–0.88	–0.25	1.98

Show that there is direct current part of 0.75 amp in variable current and obtain the amplitude of the first harmonic. [07]

SECTION - II

Q.7) (A) Trace the following curves : **(Any Two)** [08]

(a) $x^2y^2 = a^2 (y^2 - x^2)$

(b) $r = a \sin 3\theta$

(c) $x = t, \; y = t(t^2 - 1)$

(B) Verify the rule of differentiation under the integral sign for the integral [05]

$$\int_a^{a^2} \frac{1}{x+a} dx$$

(C) Find the length of the arc of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ between two consecutive cusps. [04]

OR

Q.8) (A) Trace the following curves : **(Any Two)** **[08]**

(a) $x^3 + y^3 = 3axy$

(b) $r = \frac{a}{2}(1 + \cos\theta)$

(c) $x = a\cos^3 t,\ y = a\sin^3 t$

(B) Show that $\int_0^\infty e^{-x^2-2bx}\,dx = \frac{\sqrt{\pi}}{2}[1 - \text{erf}(b)]$ **[04]**

(C) Find the length of one loop of the curve $8y^2 = x^2(1 - x^2)$. **[05]**

Q.9) (A) Find the equation of the sphere which passes through the circle $x^2 + y^2 + z^2 - 2x + 2z - 2 = 0,\ y = 0$ and touches the plane $y - z - 7 = 0$. **[06]**

(B) Find the semi-vertical angle and the equation of the right circular cone having vertex at (0, 0, 3) and passing through the circle $x^2 + y^2 = 16,\ z = 0$. **[05]**

(C) Find the equation of a right cirular cylinder of radius 4, whose axis passes through origin and makes equal angles with the co-ordinate axes. **[05]**

OR

Q.10) (A) Find the equation of the sphere which passes through the point (6, 4, 3) and meets the plane $x + 2y - 2z + 11 = 0$, in a circle of radius 4 units with centre at (1, −1, 5). **[06]**

(B) Find the equation of the right circular cylinder whose axis is the line $2(x - 1) = y + 2 = z$ and radius is 3. **[05]**

(C) Find the equation of the right circular cone generated by rotating the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ about the line $\frac{x}{-1} = y = \frac{z}{2}$. **[05]**

Q.11) (A) Evaluate $\int_{0}^{a}\int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} x\sqrt{x^2+y^2}\,dydx$ **[06]**

(B) Find the area inside the cardioide $r = 2a\,(1 + \cos\theta)$ and outside the circle $x^2 + y^2 = a^2$. **[05]**

(C) Find the C.G. of the area bounded by the curve $y^2\,(2a - x) = x^3$ and its asymptote. **[06]**

OR

Q.12) (A) Evaluate $\int_{0}^{2}\int_{0}^{\sqrt{4-y^2}} \frac{y\,dxdy}{\sqrt{(4 - x^2)(x^2 + y^2)}}$ **[06]**

(B) Find the volume enclosed between the cone $z = \sqrt{x^2 + y^2}$ and the paraboloid $z = x^2 + y^2$. **[05]**

(C) The density at any point (x, y) on a square lamina of side 'a' units, varies as the square of its distance from one of the diagonals. Show that the moment of inertia (M.I.) about the diagonal is $\frac{Ma^2}{5}$, where M is the mass of the lamina. **[06]**
