UNIVERSITY OF PUNE
MARCH 2008 EXAMINATION

Price Rs. ~~10~~ 15

Total No. of Questions—**12**]

[Total No. of Printed Pages—**8**

[3362]-101

S.E. (Civil) EXAMINATION, 2008

ENGINEERING MATHEMATICS—III

(2003 COURSE)

262

Time : Three Hours

Maximum Marks : 100

***N.B.* :—** (*i*) Answers to the two Sections should be written in separate answer-books.

(*ii*) In Section I, attempt Q. No. **1** *or* Q. No. **2**; Q. No. **3** *or* Q. No. **4**; Q. No. **5** *or* Q. No. **6**.

In Section II, attempt Q. No. **7** *or* Q. No. **8**; Q. No. **9** *or* Q. No. **10**; Q. No. **11** *or* Q. No. **12**.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Figures to the right indicate full marks.

(*v*) Use of non-programmable electronic pocket calculator is allowed.

(*vi*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Solve any *three* of the following differential equations : [12]

(*i*) $\left(D^2 - 4\right) y = \left(1 + e^x\right)^2 + 3^x$.

(*ii*) $\left(D^2 - 1\right) y = \cos x \cosh x$.

(*iii*) $\left(D^2 + 5D + 6\right) y = e^{-2x} \sec^2 x \left(1 + 2\tan x\right)$ using method of variation of parameters.

(*iv*) $\left(D^3 - D^2\right) y = 3x + xe^x$.

(*v*) $x^3y''' + 2x^2y'' + 2y = 10(x + 1/x)$.

P.T.O.

(*b*) Solve :

$$\frac{dx}{dt} - 2x - y = 0, \ \frac{dy}{dt} + x - 4y = 0$$

given $y(0) = 0$ and $x'(0) = 6$. [6]

Or

2. (*a*) Solve any *three* of the following differential equations : [12]

(*i*) $(D^4 + 6D^2 + 8) y = \sin^2 x \cos 2x$.

(*ii*) $(D^3 - 7D - 6) y = e^{2x} (1 + x)$.

(*iii*) $(D^2 - 1) y = (1 + e^{-x})^{-2}$.

(*iv*) $(D^2 + 1) y = 1 + \cot x$, using method of variation of parameter.

(*v*) $(1 + 2x)^2 y'' - 2(1 + 2x) y' - 12y = 6x$.

(*b*) Solve :

$$\frac{x^2 dx}{y^3} = \frac{y^2 dy}{x^3} = \frac{dz}{z}.$$ [6]

3. (*a*) For a strut of length '*l*' freely hinged at each end, satisfies the differential equation :

$$EI \frac{d^2 y}{dx^2} + Py = \frac{-Wl^2}{8} \sin \frac{\pi x}{l}.$$

Prove that deflection at the centre of the beam is $\frac{Wl^2}{8(Q - P)}$ and bending moment is $\frac{-Wl^2 Q}{8(Q - P)}$, where $Q = \frac{EI\pi^2}{l^2}$. [8]

(b) Solve :

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

if

(i) u is finite $\forall\ t$

(ii) $u(0\ t) = 0 \quad \forall\ t$

(iii) $u(l,\ t) = 0 \quad \forall\ t$

(iv) $u(x, 0) = u_0$ for $0 \le x \le l$, where 'l' being the length of bar. [8]

Or

4. (a) A spring stretches 1 cm under tension of 2 kg and has a negligible weight. It is fixed at one end and is attached to a weight w kg at the other. It is found that resonance occurs when an axial force "2cos $2t$" kg acts on weight. Show that when the free vibrations are died out, the forced vibrations are given by $x = ct \sin 2t$, find values of 'w' and 'c'. [8]

(b) A string is stretched and fastened to two points 'l' apart. Motion started by displacing the string in the form

$$u = a \sin \frac{\pi x}{l}$$

from which it is released at time $t = 0$. Find displacement $u(x,\ t)$ from one end, if the differential equation is given as :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}.$$ [8]

5. (*a*) Solve the following system of equations using Gauss elimination method with partial pivoting :

$$x_1 - 2x_2 + 3x_3 + 9x_4 = 5$$
$$3x_1 + 10x_2 + 4x_3 + 2x_4 = 7$$
$$11x_1 + 5x_2 + 9x_3 + 2x_4 = 13$$
$$2x_1 + 3x_2 + 7x_3 + 6x_4 = 11.$$

[8]

(*b*) Using Fourth order Runge Kutta method, evaluate $y(0.1)$ and $y(0.2)$ for the differential equation :

$$\frac{dy}{dx} = xy + y^2 \quad y(0) = 1.$$

[8]

Or

6. (*a*) Solve the following system of equation using Gauss-Seidel method :

$$10x_1 + x_2 + x_3 = 12;$$
$$2x_1 + 10x_2 + x_3 = 13;$$
$$2x_1 + 2x_2 + 10x_3 = 14.$$

[8]

(*b*) Using Adam-Bashforth method, determine $y(1.4)$ given that :

$$\frac{dy}{dx} = x^2(1 + y)$$

is tabulated as :

x	*y*
1	1
1.1	1.2
1.2	1.4662
1.3	1.8213

[8]

SECTION II

7. (*a*) The scores of two batsmen A and B in 10 innings, during a certain season are :

A	B
32	19
28	31
47	48
63	53
71	67
39	90
10	10
60	62
96	40
14	80

Find which of the two batsmen is more consistent. [7]

(*b*) A problem in Mathematics is given to two students A and B. The odds in favour of A solving the problem are 6 to 9 and against B solving the problem 12 to 10. If A and B attempt, find the probability that the problem is solved. [4]

(*c*) The first 4 moments about the value 5 are –4, 22, –117 and 560 respectively. Find the moments about the mean. [5]

Or

8. (*a*) Obtain the equations of the lines of regression, for the following data :

x	y
6	9
2	11
10	5
4	8
8	7

[7]

(*b*) A random sample of 200 screws is drawn from a population which represents the size of screws. If a sample is distributed normally with mean 3.15 cm and s.d. 0.025 cm, find the expected number of screws whose size falls between 3.12 cm and 3.2 cm.

Area corresponding to $z = 1.2$ is 0.3849

Area corresponding to $z = 2.0$ is 0.4772. [5]

(*c*) If on an average 8 ships out of 10 arrive safely at a port, find the mean and standard deviation of the number of ships arriving safely out of a total of 1600 ships. [4]

9. (*a*) The position vector of a particle at time t is :

$$\bar{r} = \cos(t-1)\,\vec{i} + \sinh(t-1)\,\vec{j} + mt^3\,\vec{k}.$$

Find the condition imposed on m by requiring that at time $t = 1$, the acceleration is normal to the position vector. [5]

(*b*) Find the directional derivative of the function $\phi = e^{2x - y - z}$ at (1, 1, 1) in the direction of the tangent to the curve $x = e^{-t}$, $y = 2\sin t + 1$, $z = t - \cos t$ at $t = 0$. [5]

(*c*) Prove the following, with usual notations (any *two*) :

(*i*) $\nabla \cdot \left(\frac{\overline{a} \times \overline{r}}{r} \right) = 0$

(*ii*) $\nabla^4 \left(r^2 \log r \right) = 6 / r^2$

(*iii*) $\nabla \times (\overline{r} \times \overline{u}) = \overline{r}(\nabla \cdot \overline{u}) - (\overline{r} \cdot \nabla)\, \overline{u} - 2\overline{u}$. [6]

Or

10. (*a*) If a particle moves along the cardioid $r = a(1 + \cos \theta)$ with constant velocity, show that $\frac{d\theta}{dt}$ is proportional to $\frac{1}{\sqrt{r}}$. [5]

(*b*) If the vector field :

$$\overline{F} = (x + 2y + az)\, \vec{i} + (bx - 3y - z)\, \vec{j} + (4x + cy + 2z)\, \vec{k}$$

is irrotational, find *a, b, c* and determine ϕ such that $\overline{F} = \nabla\phi$. [5]

(*c*) Prove the following, with usual notations (any *two*) :

(*i*) $\nabla \times (\phi\, \nabla\, \psi) = \nabla\phi \times \nabla\psi = -\nabla \times (\psi\, \nabla\, \phi)$

(*ii*) $\nabla \times \left(\overline{a} \times \left(\overline{b} \times \overline{r}\right)\right) = \overline{a} \times \overline{b}$

(*iii*) $\nabla \cdot \left[r\, \nabla \left(\frac{1}{r^n} \right) \right] = \frac{n(n-2)}{r^{n+1}}$. [6]

11. (*a*) If

$$\overline{F} = \left(2xy + 3z^2\right) \vec{i} + \left(x^2 + 4yz\right) \vec{j} + \left(2y^2 + 6xz\right) \vec{k},$$

evaluate :

$$\int_C \overline{F} \cdot d\overline{r}$$

where C is the curve $x = t$, $y = t^2$, $z = t^3$ joining the points (0, 0, 0) and (1, 1, 1). [5]

(*b*) Verify the divergence theorem for the function :

$$\overline{F} = x\vec{i} + y\vec{j} + z^2\vec{k}$$

over the surface bounded by the cone $z^2 = x^2 + y^2$ and the plane $z = 4$. [8]

(*c*) Test whether the motion specified by :

$$\overline{q} = \frac{k^2\left(x\vec{j} - y\vec{i}\right)}{x^2 + y^2} \quad (k = \text{constant})$$

is possible motion for an incompressible fluid. If so, determine the equations of the streamlines. [5]

Or

12. (*a*) Verify Stokes' theorem for :

$$\overline{F} = xy^2\vec{i} + y\vec{j} + z^2x\vec{k}$$

for the surface of the rectangular lamina bounded by $x = 0$, $y = 0$, $x = 1$, $y = 2$, $z = 0$. [8]

(*b*) Find the surfaces of equipressure in the case of steady motion of a liquid which has velocity potential $\phi = \log x + \log y + \log z$ and is under the action of force :

$$\overline{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}.$$ [5]

(*c*) If

$$\overline{F} = (y + z)\vec{i} + (z + x)\vec{j} + (x + y)\vec{k},$$

then show that curl curl curl curl $\overline{F} = \nabla^4\,\overline{F}$. [5]