Total No. of Questions—12] [Total No. of Printed Pages—8

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S.E. (Civil) (I Sem.) EXAMINATION, 2009

ENGINEERING MATHEMATICS–III

(2008 COURSE)

Time : Three Hours **Maximum Marks : 100**

N.B. :— (*i*) Answer Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4** Q. No. **5** *or* Q. No. **6** from Section I and Q. No. **7** *or* Q. No. **8**, Q. No. **9** *or* Q. No. **10** Q. No. **11** *or* Q. No. **12** from Section II.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Figures to the right indicate full marks.

(*v*) Use of electronic pocket calculator is allowed.

(*vi*) Assume suitable data if necessary.

SECTION I

1. (*a*) Solve the following (any *three*) : [12]

(*i*) $(D^2 - 4D + 3)y = x^2 e^{2x}$

(*ii*) $(D^2 + D)y = (1 + e^x)^{-1}$

(*iii*) $(D^2 - 2D + 2)y = e^x \tan x$

(by using variation of parameters method)

(*iv*) $(D^2 + 5D + 6)y = e^{-2x} \sec^2 x \,(1 + 2 \tan x)$

(*v*) $x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 3\cos(\log x)$

P.T.O.

(*b*) Solve the following simultaneous equations : [5]

$$\frac{dx}{dt} - wy = a\cos pt$$

$$\frac{dy}{dt} + wx = a\sin pt.$$

Or

2\. (*a*) Solve the following (any *three*) : [12]

(*i*) $(D^2 - 3D + 2)y = x^2 + 3$

(*ii*) $(D^2 + 1)y = \sin x \sin 2x$

(*iii*) $(D^2 + 3D + 2)y = 2e^{2x} \cos (e^{2x}) + \sin (e^{2x})$

(by using variation of parameters method)

(*iv*) $(D^2 + 2D + 1)y = e^{2x} + 3\cos x + 4$

(*v*) $(2x - 5)^2 \frac{d^2y}{dx^2} - 2(2x - 5)\frac{dy}{dx} - 12y = 8x - 7$

(*b*) Solve the following : [5]

$$\frac{dx}{x(y^2 - z^2)} = \frac{dy}{y(x^2 + z^2)} = \frac{dz}{z(x^2 + y^2)}$$

3\. (*a*) Solve $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ if :

(*i*) $u(0, t) = 0$

(*ii*) $u_x(l, t) = 0$

(*iii*) $u(x, t)$ is bounded

(*iv*) $u(x, 0) = \frac{u_0 x}{l}, 0 \le x \le l.$ [9]

(b) The deflection of a strut with one end built in ($x = 0$) and other supported and subjected to end thrust P, satisfies the equation :

$$\frac{d^2y}{dx^2} + a^2y = \frac{a^2R}{P}(l - x)$$

Given that $\frac{dy}{dx} = y = 0$ when $x = 0$

$y = 0$ when $x = l$.

Prove that $y = \frac{R}{P}\left[\frac{\sin ax}{a} - l\cos ax + l - x\right]$

where $al = \tan al$. [8]

Or

4. (a) A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $u = a\sin\left(\frac{\pi x}{l}\right)$ from which it is released at time $t = 0$. Find the displacement $u(x, t)$ from one end by using wave equation : [8]

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}.$$

(b) The differential equation of a whirling shaft, where W is the weight of the shaft and v its whirling speed is given by :

$$EI\frac{d^4y}{dx^4} - \frac{Wv^2}{g}y = W.$$

Taking the shaft of length $2l$, with the origin at the centre and short bearings at both ends, show that the maximum deflection of the shaft is given by : [9]

$$\frac{g}{2v^2}(\text{sech } al + \sec al - 2).$$

5. (*a*) Solve the following system of equations using Gauss-Jordon method : [8]

$$\begin{aligned} 2x_1 + x_2 + 2x_3 + x_4 &= 6 \\ 6x_1 - 6x_2 + 6x_3 + 12x_4 &= 36 \\ 4x_1 + 3x_2 + 3x_3 - 3x_4 &= -1 \\ 2x_1 + 2x_2 - x_3 + x_4 &= 10 \cdot \end{aligned}$$

(*b*) Use Runge-Kutta method of fourth order to obtain the numerical solution of :

$$\frac{dy}{dx} = x^2 + y^2 \quad , \quad y(1) = 1.5$$

in the interval (1, 1.2) with $h = 0.1$. [8]

Or

6. (*a*) Solve the following system of equations by the Gauss-Seidel method : [8]

$$\begin{aligned} 10x_1 + x_2 + x_3 &= 12 \\ 2x_1 + 10x_2 + x_3 &= 13 \\ 2x_1 + 2x_2 + 10x_3 &= 14 \cdot \end{aligned}$$

(*b*) Numerical solution of the differential equation : [8]

$$\frac{dy}{dx} = 2 + \sqrt{xy}$$

is tabulated as :

x	1.0	1.2	1.4	1.6
y	1.0	1.6	2.2771	3.0342

Find y at $x = 1.8$ by Milne's predictor-corrector method taking $h = 0.2$.

SECTION II

7. (*a*) The first four moments of a distribution about the values 5 are 2, 20, 40 and 50. Obtain the first four central moments, mean, standard deviation and coefficients of skewness and kurtosis. [6]

(*b*) Obtain the lines of regression for the following data : [6]

x	y
6	9
2	11
10	5
4	8
8	7

(*c*) Assuming that the diameter of 1000 brass plugs taken consecutively from machine form a normal distribution with mean 0.7515 cm and standard deviation 0.002 cm. How many of the plugs are likely to be approved if the acceptable diameter is 0.752 ± 0.004 cm ? [5]

Given :

$$A\ (2.25) = 0.4878$$
$$A\ (1.75) = 0.4599.$$

Or

8. (*a*) From a group of 10 students, marks obtained by each in two papers x and y are given as below :

x	y
23	25
28	22
42	38
17	21
26	27
35	39
29	24
37	32
16	18
46	44

Calculate Karl Pearson's coefficient of correlation. [6]

(*b*) A set of five similar coins are tossed 210 times and the result is :

No. of Heads	Frequency
0	2
1	5
2	20
3	60
4	100
5	31

Test the hypothesis that a data follows binomial distribution. [Given : $\chi^2_{5,0.05} = 11.070$]. [6]

(*c*) On an average a box containing 10 articles is likely to have 2 defectives. If we consider a consignment of 100 boxes, how many of them are expected to have 3 or less defectives ? [5]

9. (*a*) If $\bar{r} \times \frac{d\bar{r}}{dt} = 0$, then show that $\bar{r}$ has constant direction. [5]

(*b*) Find the directional derivative of $\phi = x^2y + yz^2$ at P (2, –1, 1) along the line : [4]

$$2(x - 2) = (y + 1) = (z - 1).$$

(*c*) Prove the following (any *two*) :

(*i*) $\nabla \cdot \left(\frac{\bar{a} \times \bar{r}}{r^{10}}\right) = 0$

(*ii*) $\nabla^2 \left[\nabla \cdot \left(\frac{\bar{r}}{r^2}\right)\right] = 2/r^4$

(*iii*) $\nabla^4 (e^r) = \frac{4}{r} e^r + e^r$. [8]

Or

10. (*a*) If directional derivative of $\phi = ax^2y + by^2z + cz^2x$ at P (1, 1, 1) has maximum magnitude 15 in the directional parallel to :

$$\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1},$$

find the values of a, b, c. [5]

(*b*) If a vector field

$$\overline{F} = (x + 2y + az)\,\hat{i} + (bx - 3y - z)\,\hat{j} + (4x + cy + 2z)\,\hat{k}$$

is irrotational, find a, b, c and hence determine scalar function ϕ such that $\overline{F} = \nabla\phi$. [4]

(*c*) Prove the following (any *two*) :

(*i*) For any scalar function :

$$f(r),\ \nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$$

(*ii*) If $\overline{E}$ is solenoidal, then :

curl curl curl curl $\overline{E} = 0$.

(*iii*) A vector field $\frac{\overline{r}}{r^3}$ is irrotational as well as solenoidal. [8]

11. (*a*) Show that the vector field :

$$\overline{F} = (y^2 \cos x + z^3)\,\hat{i} + (2y \sin x - 4)\,\hat{j} + (3xz^2 + 2)\hat{k}$$

is conservative and hence find the work done in moving the particle from : [5]

$$A\,(0, 1, -1) \text{ to } B\left(\frac{\pi}{2}, -1, 2\right)$$

(b) Verify Stokes' theorem for $\overline{F} = x^2 \hat{i} + xy \hat{j}$ for the surface of a triangular lamina bounded by : [6]

$$x = 1, y = 1, x + y = 3.$$

(c) Show that the velocity potential :

$$\phi = \frac{1}{2} a (x^2 + y^2 - 2z^2)$$

satisfies the equation $\nabla^2 \phi = 0$. Also determine the stream lines. [5]

Or

12. (a) Using Divergence theorem, evaluate :

$$\iint_S (y^2 z^2 \hat{i} + x^2 z^2 j + x^2 y^2 \hat{k}) . d\overline{s}$$

where S is the upper part of the sphere

$x^2 + y^2 + z^2 = 9$ above xy plane. [6]

(b) Evaluate :

$$\oint_C \left[\cos y \hat{i} + x (1 - \sin y) \hat{j}\right] . d\overline{r}$$

for a closed curve C given by : [5]

$$\frac{x^2}{9} + \frac{y^2}{4} = 1.$$

(c) Find the work done in moving the particle once around the circle $x^2 + y^2 = 16, z = 0$ if the force field is : [5]

$$\overline{F} = (2xy + 3z^2) \hat{i} + (x^3 + 4yz) j + (2y^2 + 6xz) \hat{k}.$$