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S.E. (E/TC/Instru/Comp/IT. etc.) (II Sem.) EXAMINATION, 2009

ENGINEERING MATHEMATICS—III

(2003 COURSE)

Time : Three Hours **Maximum Marks : 100**

N.B. :— (*i*) From Section **I** attempt Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**.

From Section **II** attempt Q. No. **7** *or* Q. No. **8**, Q. No. **9** *or* Q. No. **10**, Q. No. **11** *or* Q. No. **12**.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Figures to the right indicate full marks.

(*v*) Use of electronic pocket calculator and steam tables is allowed.

(*vi*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Attempt any *three* of the following : [12]

(*i*) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{1}{1+e^x}$

(*ii*) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = x^2 + \sin x$

(*iii*) $(4x+1)^2 \frac{d^2y}{dx^2} + 2(4x+1)\frac{dy}{dx} + y = 2x+1$

(*iv*) $(D^2 + 1)y = x \sin x$ (By variation of parameters method)

(*v*) $(D^3 + 4D)y = \sin 5x \cdot \cos 3x$.

(*b*) A capacitor of 10^{-3} farads is in series with an e.m.f. of 20 V and an inductor of 0.4 Henry. At $t = 0$, the charge q and current i are zero. Find q and i at time t. [4]

Or

2. (*a*) Attempt any *three* of the following : [12]

(*i*) $(D^3 + 3D^2 - 4)y = 6e^{-2x} + 4x^4$

(*ii*) $\frac{d^2y}{dx^2} - y = \cos x . \cosh x + 3^x$

(*iii*) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = \log x^2 + x - 1$

(*iv*) $\frac{dx}{z(x+y)} = \frac{dy}{z(x-y)} = \frac{dz}{x^2+y^2}$

(*v*) $(D^2 - 4D + 4)y = e^x \cdot \cos^2 x$.

(*b*) Solve : [4]

$$\frac{du}{dx} + v = \sin x$$

$$\frac{dv}{dx} + u = \cos x$$

given at $x = 0$, $u = 1$ and $v = 0$.

3. (a) Evaluate :

$$\int_C \frac{\sin \pi z^2 + 2z}{(z-1)^2 (z-2)} dz$$

where C is the circle $|z| = 4$, using the Residue theorem. [6]

(b) Find bilinear transformation which maps the points $z = 0, -1, \infty$ from the z-plane into the points $w = -1, -2 - i, i$ of w-plane. [6]

(c) Show that the Bilinear transformation $w = \frac{2z+3}{z-4}$ maps the circle $x^2 + y^2 - 4x = 0$ into the line $4u + 3 = 0$. [4]

Or

4. (a) If $w = \phi + i\psi$ represents the complex potential for an electric field and $\psi = x^2 - y^2 + \frac{x}{x^2 + y^2}$, determine the function ϕ. Also find w in terms of z. [6]

(b) Evaluate :

$$\int_C \frac{\sin 2z}{\left(z + \frac{\pi}{3}\right)^4} dz,$$

where C is $|z| = 2$. [5]

(c) Show that an analytic function $f(z)$ with constant modulus is constant. [5]

5. (*a*) Find the *z*-transform (any *two*) : [6]

(*i*) $f(k) = \begin{cases} -(-1/4)^k & , \quad k < 0 \\ (-1/5)^k & , \quad k \geq 0 \end{cases}$

(*ii*) $f(k) = \sin\left(\frac{k\pi}{4} + \alpha\right), k \geq 0, \alpha$ constant

(*iii*) $f(k) = k^2 e^{-ak}, k \geq 0.$

(*b*) Obtain the Fourier integral representation of : [8]

$$f(x) = \begin{cases} 1 - x^2 & , \quad |x| \leq 1 \\ 0 & , \quad |x| > 1 \end{cases}$$

Hence evaluate :

$$\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right) \cos\frac{x}{2}\, dx.$$

(*c*) Solve the following integral equation : [4]

$$\int_0^\infty f(u) \sin \lambda u\, du = \begin{cases} 1 - \lambda & , \quad 0 \leq \lambda \leq 1 \\ 2 & , \quad 1 < \lambda \leq 2 \\ 0 & , \quad \lambda > 2 \end{cases}.$$

Or

6. (*a*) Find the Fourier cosine transform of : [6]

$$f(x) = e^{-ax^2}, a > 0, x > 0.$$

(*b*) Find the inverse *z*-transform (any *two*) : [8]

(*i*) $F(z) = \frac{z(z+1)}{(z-1)^2}, |z| > 1$

(*ii*) $F(z) = \dfrac{z^2}{\left(z - \frac{1}{4}\right)\left(z - \frac{1}{5}\right)}, \dfrac{1}{5} < |z| < \dfrac{1}{4}$

(*iii*) $F(z) = \dfrac{z^2}{z^2 + 1}$ (using Inversion Integral Method).

(*c*) Solve the difference equation : [4]

$$f(k+1) - f(k) = 1, \; f(0) = 0.$$

SECTION II

7. (*a*) Find Laplace transforms of the following (any *two*) : [8]

(*i*) $f(t) = t^2 e^{-t} \sin^3 t$

(*ii*) $f(t) = \begin{cases} t & 0 < t < 4 \\ 5 & t > 4 \end{cases}$

(*iii*) $\dfrac{\cos at - \cos bt}{t}$.

(*b*) Prove that : [4]

$$\delta(t-a) * \delta(t-b) = \delta(t-a-b).$$

(*c*) Solve, using Laplace transform method : [5]

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = t\,e^{-t} \text{ with } y(0) = 1, \; y'(0) = -2.$$

Or

8. (*a*) Find inverse Laplace transforms of the following (any *two*): [8]

(*i*) $\dfrac{1}{(s-2)^4 (s+3)}$

(ii) $\dfrac{2s+5}{s^2+4s+13}$

(iii) $\dfrac{e^{-2s}}{\sqrt{s+5}}$.

(b) Using Laplace transform technique, prove that : [4]

$$B(m,n) = \frac{\overline{|m}\,\overline{|n}}{\overline{|m+n}}.$$

(c) Solve the following, by using Laplace transform : [5]

$$\frac{d^2x}{dt^2} + 9x(t) = 18t \quad \text{with } x(0) = 0,\ x\left(\frac{\pi}{2}\right) = 0.$$

9. (a) Show that tangent at any point on the curve :

$$x = e^{\theta} \cos\theta,\ y = e^{\theta} \sin\theta,\ z = e^{\theta}$$

makes constant angle with z-axis. [5]

(b) Show that :

$$\overline{F} = (6xy + z^3)\,\overline{i} + (3x^2 - z)\,\overline{j} + (3xz^2 - y)\,\overline{k}$$

is irrotational. Find scalar ϕ such that $\overline{F} = \nabla\Phi$. [5]

(c) Find directional derivative of :

$$\Phi = xy^2 + yz^3 + zx^2$$

at the point (1, 1, 1) along a line equally inclined with coordinate axes. [6]

Or

10. (*a*) Establish the following : [6]

(*i*) $$\nabla \times \left(\frac{\bar{a} \times \bar{r}}{r^3}\right) = \frac{-\bar{a}}{r^3} + \frac{3(\bar{a}.\bar{r})}{r^5}\bar{r}$$

(*ii*) $$\nabla^2 f(r) = \frac{d^2 f}{dr^2} + \frac{2}{r}\frac{df}{dr}.$$

(*b*) With usual notations, show that $\overline{\mathrm{F}} = (\bar{a}.\bar{r})\,\bar{a}$ is irrotational and find corresponding scalar function ϕ. [5]

(*c*) If the directional derivative of $\phi = axy + byz + czx$ at (1, 1, 1) has maximum magnitude 4 in a direction parallel to z-axis, find the values of a, b, c. [5]

11. (*a*) Evaluate :

$$\int_{\mathrm{C}} \overline{\mathrm{F}}.\,d\bar{r}$$

for $\overline{\mathrm{F}} = 3x^2\bar{i} + (2xz - y)\,\bar{j} + z\bar{k}$

along the path $x^2 = 4y$, $3x^3 = 8z$ from $x = 0$ to $x = 2$. [5]

(*b*) Verify Divergence theorem for $\overline{\mathrm{F}} = 4xz\,\bar{i} - y^2\bar{j} + yz\,\bar{k}$ and S is the surface of the cube bounded by $x = 0$, $x = a$, $y = 0$, $y = a$ and $z = 0$, $z = a$. [6]

(c) Evaluate :

$$\iint_S (\nabla \times \overline{F}).\hat{n}\ ds$$

where $\overline{F} = (x-y)\overline{i} + (x^2 + yz)\overline{j} - 3xy^2\,\overline{k}$

and S is the surface of the cone $z = 4 - \sqrt{x^2 + y^2}$ above $x - y$ plane. [6]

Or

12. (a) Evaluate the surface integral :

$$\iint_S (y^2z^2\overline{i} + z^2x^2\overline{j} + x^2y^2\overline{k}).d\overline{s}$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ in the positive octant. [6]

(b) Apply Stokes theorem to evaluate :

$$\int_C 4y\,dx + 2z\,dy + 6y\,dz$$

where 'C' is the curve of intersection of :

$x^2 + y^2 + z^2 = 6z$ and $z = x + 3$. [6]

(c) Maxwell's equations are given by :

$$\nabla.\overline{E} = 0,\ \ \nabla.\overline{H} = 0,\ \ \nabla \times \overline{E} = -\frac{\partial \overline{H}}{\partial t},\ \ \nabla \times \overline{H} = \frac{\partial \overline{E}}{\partial t}$$

show that :

$\overline{E}$ and $\overline{H}$ satisfy $\nabla^2 u = \dfrac{\partial^2 u}{\partial t^2}$. [5]