Total No. of Questions—**12**] [Total No. of Printed Pages—**8+4+1**

[3662]-201

S.E. (Computer Engg.) (I Sem.)

(Common to IT) EXAMINATION, 2009

DISCRETE STRUCTURES

(2008 COURSE)

Time : Three Hours **Maximum Marks : 100**

N.B. :— (*i*) Answers to the two Sections should be written in separate answer books.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Show that :

(*i*) $(p \wedge (\sim p \vee q)) \vee (q \wedge \sim(p \wedge q))$ is equivalent to q.

(*ii*) $((p \vee \sim q) \wedge (\sim p \vee \sim q)) \vee q$ is a tautology. [8]

(*b*) The converse of statements is given. Write inverse and contrapositive statements :

(*i*) If he is considerate of others, then a man is a gentleman.

(*ii*) If a steel rod is stretcher, then it has been heated. [4]

P.T.O.

(*c*) Show that ~p follows logically from

$$\sim(p \wedge \sim q),\ \sim q \vee p,\ \sim r.$$ [4]

Or

2. (*a*) Test the validity of argument.

If a person is poor, he is unhappy.

If a person is unhappy, he dies young,

therefore poor person dies young. [2]

(*b*) Among 100 students, 32 study mathematics, 20 study physics, 45 study biology, 15 study mathematics and biology, 7 study mathematics and physics, 10 study physics and biology and 30 do not study any of three subjects.

(*i*) Find the number of students studying all three subjects.

(*ii*) Find the number of students studying exactly one of the three subjects. [8]

(*c*) Prove that

$$n < 2^n \text{ for } n > 1$$

using mathematical induction. [6]

3. (*a*) Consider the binary operation * on Q, the set of rational numbers defined by

$$a * b = a + b - ab \quad \forall\ a,\ b \in Q$$

determine whether * is associative. [6]

(*b*) Consider an algebraic system (Q, *), where Q is set of rational numbers and * is binary operation defined by

$$a * b = a + b - ab \quad \forall\ a,\ b \in Q.$$

Determine whether (Q, *) is a group. [6]

(*c*) Define rings and state types of rings with examples. [4]

Or

4. (*a*) Show that :

$$S = \left\{a + b\sqrt{2};\ a;\ b \in z\right\}$$

for operation +, * is an integral domain. [4]

(*b*) Given a parity check matrix

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}.$$

Find minimum distance of code generated by H. How many errors can it detect and correct ? [6]

(*c*) Find the roots of

$$f(t) = 2t^4 - 11t^3 + 33t^2 - 19t - 65.$$

Given that $t = 2 + 3i$ is one root. [6]

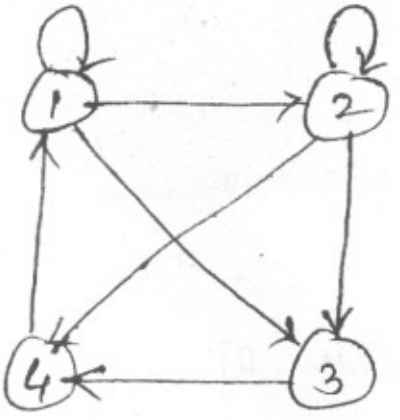
5. (*a*) Let R and S be two relations whose corresponding diagraphs are shown in figure below. Compute :

(*i*) $\bar{R}$

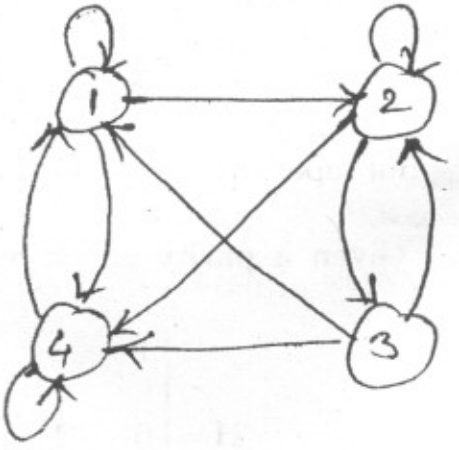
(*ii*) $R \cap S$

(*iii*) $R \cup S$

(*iv*) S^{-1}. [4]



R



S

(*b*) Let

$$A = \{1, 2, 3, 4\}$$

and let R and S be the relations on A described by

$$M_R = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad M_S = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$

Use Warshall's algorithm to compute the transitive closure of $R \cup S$. [6]

(*c*) Show that 7 colours are used to paint 50 bicycles, at least 8 bicycles will be of same colour. [4]

(*d*) Define chain and antichain with example. [4]

Or

6. (*a*) Find discrete numeric function corresponding to the following generating function : [6]

$$\frac{1}{5 - 6z + z^2}.$$

(*b*) Let f, g, h be the function from N to N, where N is the set of natural numbers so that

$$f(n) = n + 1,\ g(n) = 2n,$$

$$h(n) = \begin{cases} 0 & n \text{ is even} \\ 1 & n \text{ is odd.} \end{cases}$$

Determine $f \circ f$, $f \circ g$, $g \circ f$, $g \circ h$, $h \circ g$, $(f \circ g) \circ h$. [6]

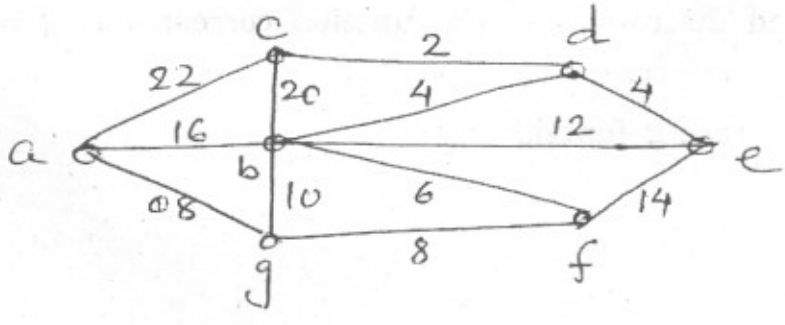
(*c*) Let R be a binary relation on the set of all positive integers such that :

$R = \{(a, b)/(a - b)$ is an odd positive integer$\}$

Is R reflexive ? Symmetric ? Antisymmetric ? Transitive ? An equivalent relation ? A partially ordered relation ? [6]

SECTION II

7. (*a*) Find the shortest path for a to e in the following graph : [6]



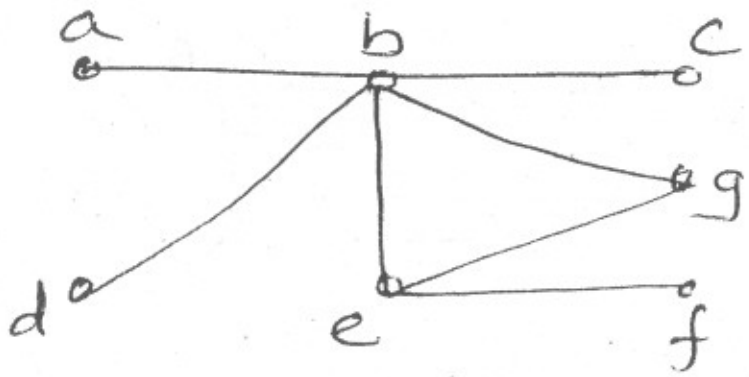
(*b*) Find number of edges in graph with 6 nodes two of edge of degree 4 and four of degree 2, draw such graph. [6]

(*c*) How many edges has each of the following ? [3]

(*i*) K_{10}

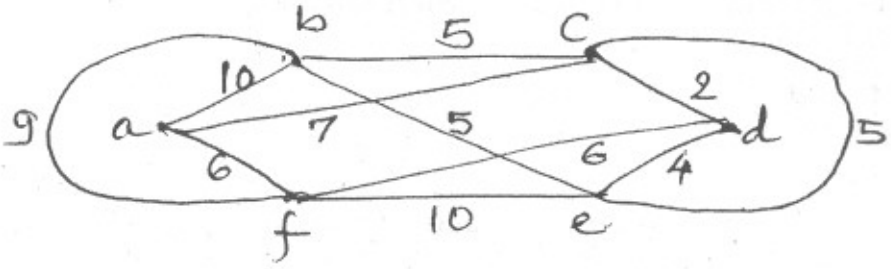
(*ii*) $K_{5,\ 7}$.

(*d*) Define cut-sets/points. Find all cut-points of the following graph : [3]



Or

8. (*a*) The nearest neighbour method to find Hamiltonian circuit starting from *a*, find its weight. [6]



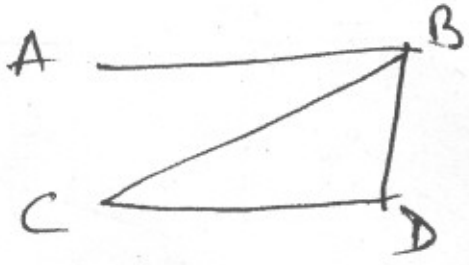
(*b*) Define subgraph and full subgraph, determine whether

$$H = H' = (V', E')$$

is subgraph of G where :

(*i*) $V' = \{A, B, F\}$, $E' = \{(A, B), (A, F)\}$

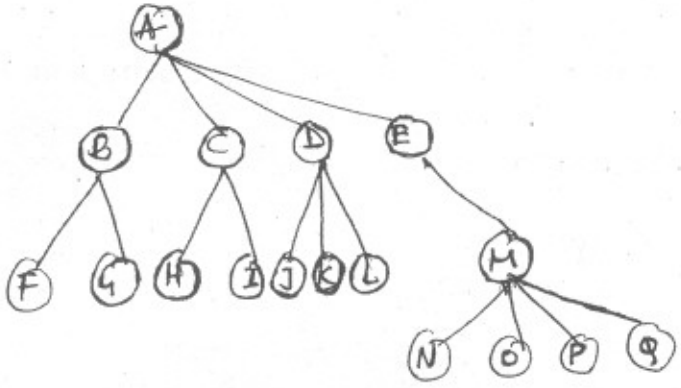
(*ii*) $V' = \{B, C, D\}$, $E' = \{(B, C), (B, D)\}$. [6]



(*c*) Find whether the complete graph K_6 and complete bipartite graph $K_{3,3}$ are isomorphic or not. [3]

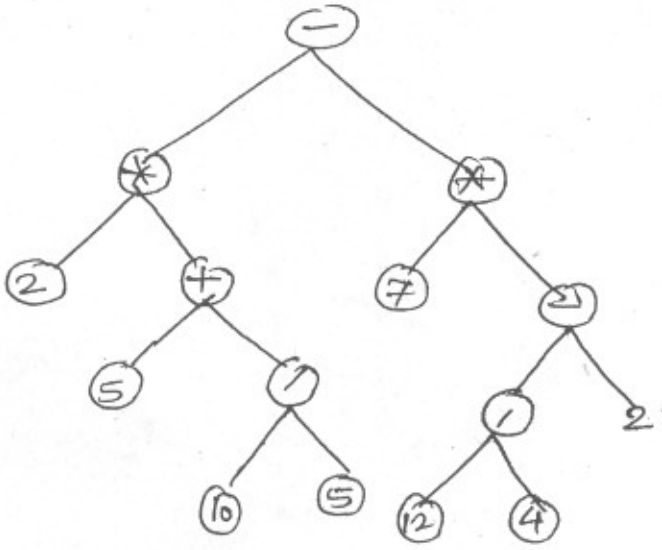
(*d*) Give an example of graph that has Euler circuit and Hamiltonian circuit which are distinct, draw graph and give circuit. [3]

9. (*a*) Define binary tree and convert the following tree into binary tree : [6]



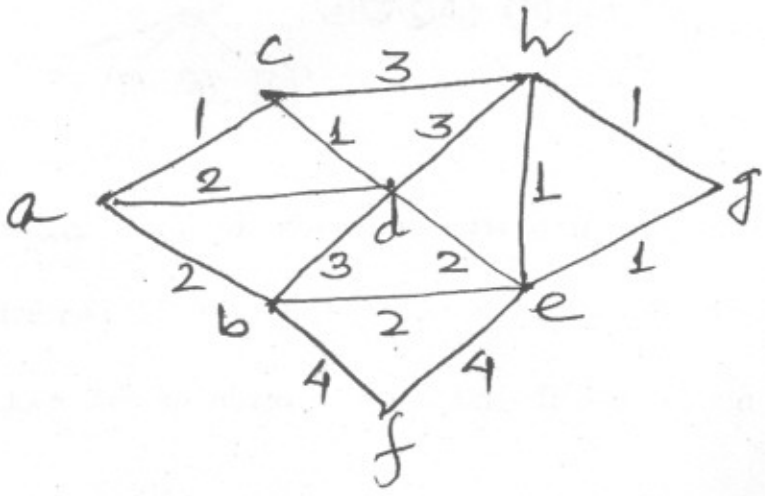
(*b*) A binary search tree generated by inserting integers in order 50, 15, 62, 5, 20, 58, 91, 3, 8, 37, 60, 24. Determine the number of nodes in left and right subtree of the root. [6]

(*c*) Write and evaluate the expression tree shown below : [4]

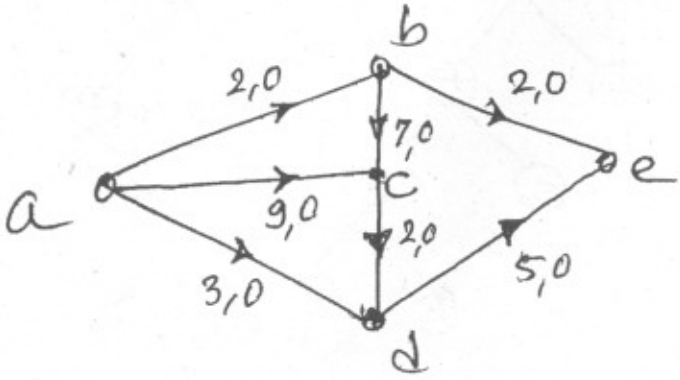


Or

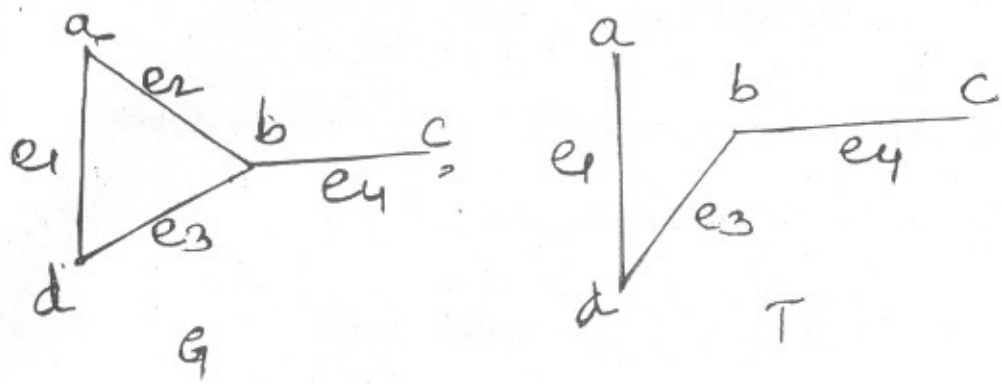
10. (*a*) Using Prims algorithm, construct a minimal spanning tree of weighted graph starting at vertex *a*, repeat for vertex *b*. Verify if both tree have same weight. [6]



(*b*) Find max. flow in transport network as shown in figure below : [6]



(c) Find fundamental cut-sets of graph w.r.to spanning tree T as shown below : [4]



11. (a) Suppose license plate contains 3 English letter followed by 4 digits :

(i) How many different license plates can be manufactured if repetition of letters and digits are allowed ?

(ii) How many plates are possible if only the letters are repeated ?

(iii) How many plates are possible if only the digits are repeated ? [6]

(b) There are radar, a computer and gyroscope on board of an aeroplane. The probability that the radar fails is 0.2, if the radar fails, the gyroscope will also fails. The probability that computer

fails is 0.3. If radar functions correctly and the probability that the gyroscope fails is 0.2.

(*i*) Describe sample space.

(*ii*) What is probability that the computer or gyroscope functions correctly while other doesn't ?

(*iii*) What is the probability that the radar functions correctly if one of the other two system fails ? [6]

(*c*) Find number of ways of arranging the letters of the word TENNESSEE all at a time (*i*) if there is no restriction (*ii*) if the first two letters must be 'E'. [4]

Or

12. (*a*) Suppose repetitions are permitted :

(*i*) How many ways three digit number can be formed from six digits 2, 3, 4, 5, 7 and 9 ?

(*ii*) How many of these numbers are less than 400 ?

(*iii*) How many are even ?

(*iv*) How many are odd ?

(*v*) How many are multiple of 5 ?

(*vi*) How many are multiple of 10 ? [6]

(*b*) Out of 4 officers and 10 clerks, a committee of 2 officers and 3 clerks is to be formed. In how many ways committee can done if :

(*i*) any officer and any clerk can be included.

(*ii*) a particular clerk must be in committee.

(*iii*) a particular officer can not be in committee. [6]

(*c*) 12 persons are made to sit around a table. Find the number of ways they can sit such that 2 specific persons are not together. [4]