Total No. of Questions—**12**] [Total No. of Printed Pages—**8+4**

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S.E. (Compu. Engg. I.T.) EXAMINATION, 2009

DISCRETE STRUCTURES

(2003 COURSE)

Time : Three Hours **Maximum Marks : 100**

***N.B.* :—** (*i*) Answer *three* questions from Section I and *three* questions from Section II.

(*ii*) Answers to the two Sections should be written in separate answer books.

(*iii*) Figures to the right indicate full marks.

SECTION I

1. (*a*) Show that :

$$1^2 + 2^2 + 3^2 + \dots\dots\dots + n^2 = n(n + 1)(2n + 1)/6, \; n \geq 1$$

by mathematical induction. [6]

(*b*) Among 120 mathematics students at a college concerning the languages French, German and Russian :

65 study French, 45 study German, 42 study Russian, 20 study French and German, 25 study French and Russian, 15 study German and Russian, 8 study all three languages.

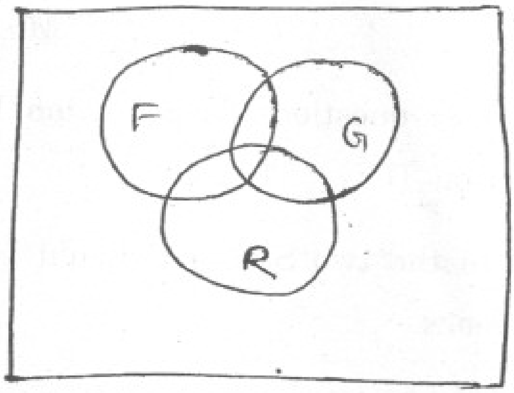
P.T.O.

Let F, G and R denote the sets of students studying French, German and Russian respectively.

Find the number of students who study :

(*i*) at least one language.

(*ii*) only one language. [8]



(*c*) Let :

$$A = \{a, b, \{a, b\}, \{\{a, b\}\}\}$$

identify each of the following statements as True or False. Justify your answers :

(*i*) $a \in A$

(*ii*) $\{a\} \in A$

(*iii*) $\{a, b\} \in A$

(*iv*) $\{\{a, b\}\} \subseteq A$. [4]

Or

2. (*a*) Obtain the conjunctive normal form and disnjunctive normal form for :

(*i*) $p \leftrightarrow (\overline{p} \vee \overline{q})$

(*ii*) $(p \vee \overline{q}) \rightarrow q$ [6]

(*b*) Write the following statements in symbolic form using quantifiers :

(*i*) All students have taken a course in communication skills.

(*ii*) There is a girl student in the class who is also a sports-person.

(*iii*) Some students are intelligent, but not hardworking. [6]

(*c*) Using Venn diagram, prove or disprove :

(*i*) $A \oplus (B \oplus C) = (A \oplus B) \oplus C$

(*ii*) $A \cap B \cap C = A - [(A - B) \cup (A - C)]$. [6]

3. (*a*) Define the terms with examples :

(*i*) Rule of sum and rule of product.

(*ii*) Discrete probability and conditional probability.

(*iii*) Binomial distribution. [6]

(*b*) When a certain defective die is tossed, the numbers from 1 to 6 will appear with the following probabilities :

$$P(1) = \frac{2}{18} \quad P(2) = \frac{3}{18} \quad P(3) = \frac{4}{18}$$

$$P(4) = \frac{3}{18} \quad P(5) = \frac{4}{18} \quad P(6) = \frac{2}{18}$$

Find the probability that :

(*i*) an odd number is on top.

(*ii*) a prime number is on top.

(*iii*) a number less than 5 is on the top.

(*iv*) a number greater than 3 is on the top. [8]

(*c*) Consider an experiment of tossing a coin three times. What is the probability of getting exactly one head ? [2]

Or

4. (*a*) 6 boys and 6 girls are to be seated in a row. How many ways can they be seated if :

(*i*) All boys are to be seated together and all girls are to be seated together.

(*ii*) No two girls should be seated together.

(*iii*) Boys occupy extreme positions. [6]

(*b*) A bag contains 6 white marbles and 5 red marbles. Find the number of ways four marbles can be drawn from the bag if :

(*i*) They can be of any colour.

(*ii*) Two must be white and two red.

(*iii*) They must be all of the same colour. [6]

(*c*) Five gentlemen attend a party, where before joining the party, they leave their overcoats in a checkroom. After the party, the overcoats get mixed up are returned to the gentlemen in a random manner. Find the number of ways in which none receives his own overcoat. [4]

5. (*a*) Define equivalence relation.

Consider the following five relations on the set :

$A = \{1, 2, 3, 4\}$

$R_1 = \{(1, 1), (1, 2), (2, 3), (1, 3), (4, 4)\}$

$R_2 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$

$R_3 = \{(1, 3), (2, 1)\}$

$R_4 = \phi$, the empty relation

$R_5 = A \times A$, the universal relation.

Determine which of the relations are reflexive and symmetric. [6]

(b) Let $f : z \to z$ be defined as $f(x) = x^2 + 2x + 2$ and $g : z \to z$ be defined as $g(x) = x - 1$, find :

(i) $g \circ f$

(ii) $f \circ g$

(iii) $f \circ f$

(iv) $g \circ g$. [4]

(c) Determine $a * b$ for the following numeric functions

$$a_r = \begin{cases} 1, & 0 \le r \le 2 \\ 0, & r \ge 3 \end{cases} \quad \text{and}$$

$$b_r = \begin{cases} r + 1, & 0 \le r \le 2 \\ 0, & r \ge 3 \end{cases}$$ [6]

Or

6. (a) Write Warshall's algorithm and find the transitive closure of R by Warshall's algorithm, where

$$A = \{1, 2, 3, 4, 5, 6\} \text{ and}$$

$$R = \{(x, y) / |x - y| = 2\}.$$ [8]

(*b*) Define :

(*i*) Functions.

(*ii*) Onto functions.

(*iii*) One-to-one functions.

(*iv*) One-to-one-onto functions. [4]

(*c*) If

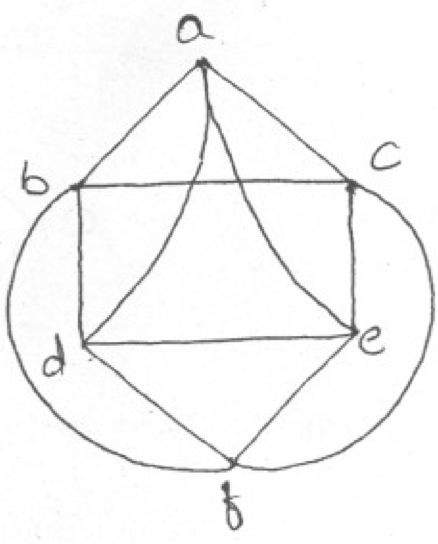
A = {1, 2, 3, 4}

R = {(1, 1), (1, 2), (2, 2), (2, 4), (1,3), (3, 3), (3, 4), (1, 4), (4, 4)}

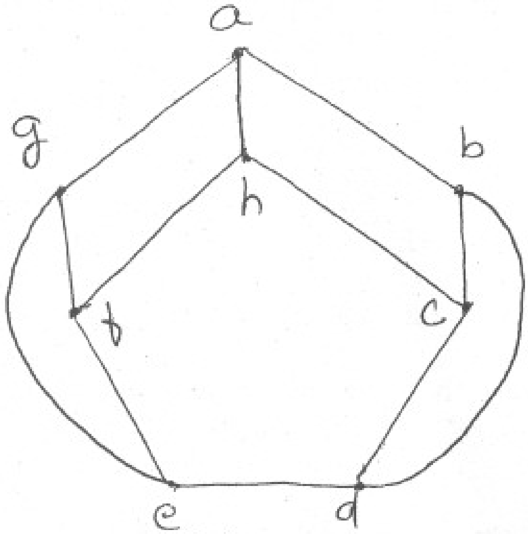
then show that R is a partial order and draw its Hase diagram. [4]

SECTION II

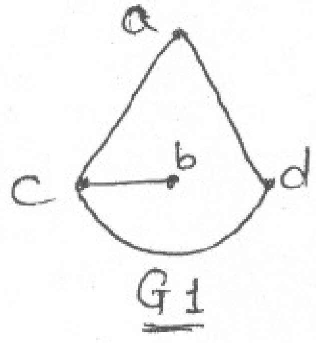
7. (*a*) For the graph given in figure, find whether it contains Eulerian circuit or not. If yes, justify your answer. [6]

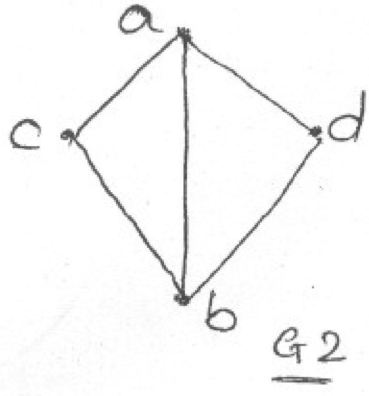


(b) For the graph below, find whether it contains Hamiltonian paths and circuit. If yes, justify your answer. [6]



(c) Find whether the following pairs of graphs are isomorphic or not. Justify. [6]





Or

8. (*a*) How many edges must a planar graph have if it has 7 regions and 5 nodes ? Draw one such graph. [6]

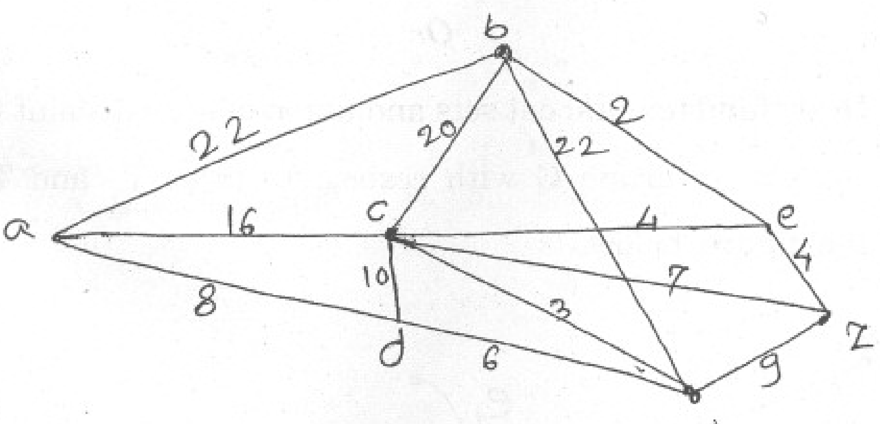
(*b*) Explain with example : [6]

(*i*) Spanning subgraph

(*ii*) Path and circuit in a graph

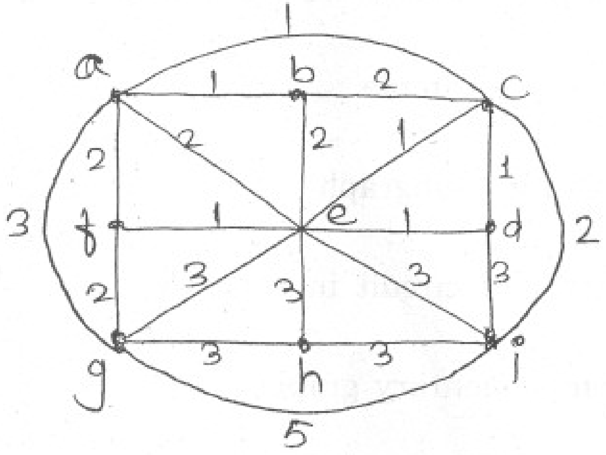
(*iii*) Complementary graph.

(*c*) Apply the shortest path algorithm and find shortest path between *a*-*z* in the graph given below : [6]



9. (*a*) Construct an optimal tree for weights 8, 9, 10, 11, 13, 15, 22. Draw all the steps. [6]

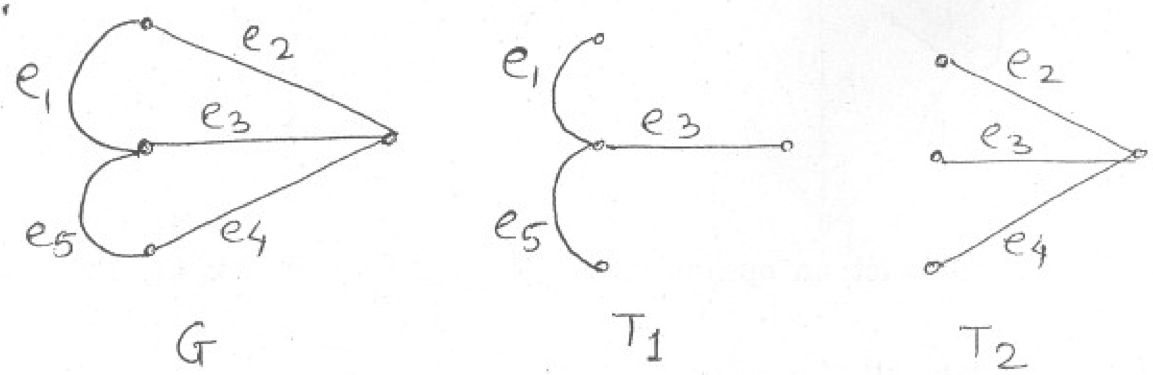
(*b*) Find minimum spanning tree using Prim's algorithm for the graph given below : [6]



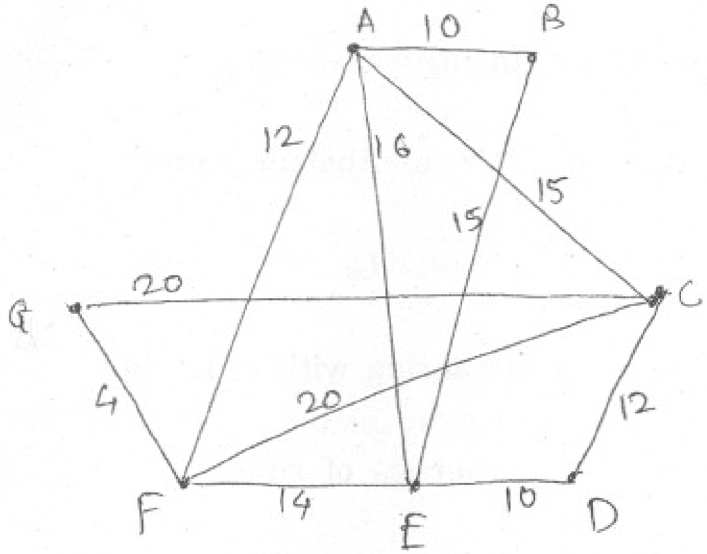
(*c*) What is transport network ? Explain the terms maximum flow, cut and capacity of a cut. [4]

Or

10. (*a*) Draw fundamental cut sets and union of edge disjoint fundamental cut sets of graph G with respect to trees T_1 and T_2 shown in the figure below. [8]



(*b*) Give the stepwise construction of minimum spanning tree for the following graph, using Kruskal's algorithm. [6]



(*c*) Explain fundamental system of cutset. [2]

11. (*a*) Define :

(*i*) Monoid

(*ii*) Submonoid

with example. [4]

(*b*) Let A = {*a*, *b*}. Which of the following define a semigroup on A. Which define a monoid on A ? Justify. [8]

(*i*)

*	*a*	*b*
a	*a*	*b*
b	*a*	*a*

(*ii*)

*	*a*	*b*
a	*a*	*b*
b	*b*	*b*

(*c*) Let (A, *) be a monoid such that for every X in A,

$$X * X = e,$$

where e is the identity element.

Show that (A, *) is an Abelian group. [4]

Or

12. (*a*) Explain ring and subring with example. [4]

(*b*) What are the properties of ring ? [4]

(*c*) Consider the set Z together with binary operation $\oplus$ and $\odot$ which are defined by :

$$X \oplus Y = X + Y - 1$$

$$X \odot Y = X + Y - XY$$

then prove that $(Z, \oplus, \odot)$ is a ring. [8]