Total No. of Questions : 12] [Total No. of Printed Pages : 7

[3861]-151

F. E. (Semester - I) Examination - 2010

ENGINEERING MATHEMATICS - I

(2008 Pattern)

Time : 3 Hours] [Max. Marks : 100

Instructions :

(1) *Answers to the* ***two sections*** *should be written in* ***separate answer books****.*

(2) *Black figures to the right indicate full marks.*

(3) *Neat diagrams must be drawn wherever necessary.*

(4) *Assume suitable data, if necessary.*

(5) *Use of electronic pocket calculator is allowed.*

SECTION - I

Q.1) (A) Reduce the following matrix A to its normal form and hence find its rank, where **[05]**

$$A = \begin{bmatrix} 2 & -1 & 1 & 3 \\ 2 & 4 & -1 & 0 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

(B) Examine the consistency of the system of the following equations. If consistent, solve system of the equations : **[06]**

$x + y - z + t = 2$

$2x + 3y + 4t = 9$

$y - 2z + 3t = 2$

(C) Verify Cayley Hamilton Theorem for the matrix **[07]**

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

OR

Q.2) (A) Find Eigen Values and corresponding Eigen Vectors for the matrix **[07]**

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

(B) Examine whether the following vectors are linearly dependent. If so, find the relation between them :

$X_1 = (2, -2, 4)$, $X_2 = (-1, 3, -3)$, $X_3 = (1, 1, 1)$ **[05]**

(C) Find values of a, b, c so that the matrix

$$A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$$

becomes an orthogonal matrix. **[06]**

Q.3) (A) If $\frac{Z-1}{Z+i}$ is a purely imaginary number, then show that the locus of Z is a circle. **[06]**

(B) Show that the continued product of all values of $(1+i\sqrt{3})^{\frac{1}{4}}$ is $2\left[\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right]$. **[05]**

(C) If α, β are roots of an equation,

$\sin^2\theta\ z^2 - (2\sin\theta\cos\theta)\ z + 1 = 0$, prove that

$\alpha^n + \beta^n = 2\cos n\theta\ \text{cosec}^n\theta$, where n is an integer. **[05]**

OR

Q.4) (A) Find tanhx if $5 \sinh x - \cosh x = 5$ **[05]**

(B) If $u + iv = \sin(x + iy)$,

prove that :

(a) $u^2\text{cosec}^2x - v^2\sec^2x = 1$

(b) $u^2\text{sech}^2x + v^2\text{cosech}^2x = 1$ **[05]**

(C) A square lies above real axis in Argand's diagram and has two of its vertices at origin and the point $3 + 2i$. Find the rest two vertices of the square. **[06]**

Q.5) (A) If $y = \dfrac{x^3}{x^2 - 1}$,

then find n^{th} order differential coefficient of y w.r. to x. **[05]**

(B) If $y = \sin^{-1}[3x - 4x^3]$,

prove that $(1 - x^2)\ y_{n+2} - (2n + 1)\ xy_{n+1} - n^2y_n = 0$. **[05]**

(C) Test convergence of the series : **(Any One)** **[06]**

(a) $1 + \dfrac{1}{2} + \dfrac{1 \cdot 3}{2 \cdot 4} + \dfrac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} + \ldots\ldots$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(n + 1)\, x^n}{n^2}$

OR

Q.6) (A) If $y = (2x + 1) \log (4x + 3)$,
then find y_{20}. **[05]**

(B) If $y = \left[x + \sqrt{x^2 - 1}\right]^m$,

prove that $(x^2 - 1) y_{n+2} + (2n + 1) xy_{n+1} + (n^2 - m^2) y_n = 0$. **[05]**

(C) Test convergence of the series : **(Any One)** **[06]**

(a) $\frac{1}{1^2 + m} + \frac{2}{2^2 + m} + \frac{3}{3^2 + m} + \ldots\ldots$

(b) $\sum_{n=1}^{\infty} \frac{4.7.10 \ldots\ldots (3n+1)}{1.2.3.4 \ldots\ldots n}$

SECTION - II

Q.7) (A) Expand $\frac{x}{e^x - 1}$ upto x^4. **[05]**

(B) Use Taylor's Theorem to obtain approximate value of $\sqrt{10}$ to four decimal places. **[05]**

(C) Solve : **(Any One)** **[06]**

(a) Find a and b, if

$$\lim_{x \to 0} \frac{a\sin^2 x + b\log\cos x}{x^4} = -\frac{1}{2}.$$

(b) Evaluate $\lim_{x \to 0} \left(\sin^2 \frac{\pi}{2 - ax}\right)^{\sec^2 \frac{\pi}{2-bx}}$

OR

Q.8) (A) Expand $\sin^{-1}x$ in ascending powers of x. **[05]**

(B) Expland $3x^2 - 2x^2 + x - 4$ in powers of $(x + 2)$. **[05]**

(C) Solve : **(Any One)** **[06]**

(a) Evaluate $\lim_{x\to 0} \dfrac{\tanh x - 2\sin x + x}{x^5}$

(b) Evaluate $\lim_{x\to 0} \dfrac{e^x \sin x - x - x^2}{x^2 + x\log(1-x)}$

Q.9) Solve : **(Any Two)** **[16]**

(A) Find value of n so that $u = r^n(3\cos^2\theta - 1)$

satisfies $\dfrac{\partial}{\partial r}\left(r^2 \dfrac{\partial u}{\partial r}\right) + \dfrac{1}{\sin\theta} \cdot \left(\sin\theta \dfrac{\partial u}{\partial \theta}\right)$.

(B) If $ux + vy = 0$; $\dfrac{u}{x} + \dfrac{v}{y} = 1$,

show that : $\left(\dfrac{\partial v}{\partial y}\right)_x - \left(\dfrac{\partial u}{\partial x}\right)_y = \dfrac{x^2 + y^2}{x^2 - y^2}$

(C) If $x = e^u \operatorname{cosec} v$; $y = e^u \cot v$, then

show that : $\left(\dfrac{\partial z}{\partial x}\right)^2 - \left(\dfrac{\partial z}{\partial y}\right)^2 = e^{-2u}\left[\left(\dfrac{\partial z}{\partial u}\right)^2 - \sin^2 v\left(\dfrac{\partial z}{\partial v}\right)^2\right]$

OR

Q.10) Solve : **(Any Two)** **[16]**

(A) If $u = (2x + 3y)^n + \dfrac{1}{(x - y)^n}$,

show that : $x^2 u_{xx} + 2xy\, u_{xy} + y^2 u_{yy} + x u_x + y u_y = n^2 u$.

(B) If $x^3 + y^3 - 3axy = 0$,

show that : $\dfrac{d^2y}{dx^2} + \dfrac{2a^3\, xy}{\left(y^2 - ax\right)^3} = 0$.

(C) If $f(xy^2, z - 2x) = 0$,

prove that : $x \dfrac{\partial z}{\partial x} - \dfrac{y}{2} \dfrac{\partial z}{\partial y} = 2x$.

Q.11) (A) The area of ΔABC is calculated using the formula $\Delta = \dfrac{1}{2} ab\sin C$. Errors of 2%, 3%, 4% are made in measuring a, b, C respectively. If the correct value of C is 30°, find % error in the calculated value of Δ. **[06]**

(B) If $x = u + v$; $y = v^2 + w^2$; $z = w^3 + u^3$,

show that : $\dfrac{\partial u}{\partial x} = \dfrac{vw}{vw + u^2}$. **[06]**

(C) Find Stationary values of $u = x + y + z$ if $xy + yz + zx = 3a^2$. **[06]**

OR

Q.12) (A) Verify $JJ' = 1$ for $x = e^u \tan v$ and $y = e^u \sec v$. **[06]**

(B) Examine for functional dependence/independence. If dependent, find relation between them :

$$u = \frac{x - y}{x + a} \quad ; \quad v = \frac{x + a}{y + a}$$ **[06]**

(C) The sum of three positive numbers is 'a'. Determine maximum value of their product. **[06]**
