[3861]-157

F. E. (Semester - II) Examination - 2010

ENGINEERING MATHEMATICS - II

(2008 Pattern)

Time : 3 Hours] [Max. Marks : 100

Instructions :

(1) In section I, solve Q. No. 1 or No. 2, Q. No. 3 or Q. No. 4, Q. No. 5, or Q. No. 6 and In section - II, solve Q. No. 7 or Q. No. 8, Q. No. 9 or Q. No. 10, Q. No. 11 or Q. No. 12.

*(2) Answers to the **two sections** should be written in **separate books**.*

(4) Black figures to the right indicate full marks.

(5) Assume suitable data, if necessary.

SECTION - I

Q.1) (A) Form the differential equation whose general solution is $Ax^2 + By^2 = 1$ (A, B are arbitrary constants). **[05]**

(B) Solve : **(Any Three)** **[12]**

(a) $(x + y)^2 \left(x \dfrac{dy}{dx} + y\right) = xy \left(1 + \dfrac{dy}{dx}\right)$

(b) $(x + 2y - 3)\, dx - (3x + 6y - 1)\, dy = 0$

(c) $y\log y\, dx + (x - \log y)\, dy = 0$

(d) $\dfrac{dy}{dx} = - e^{x-y} \left(e^{x} + e^{y}\right)$

OR

Q.2) (A) Form the differential equation whose general solution is $y = e^x (c_1 \cos x + c_2 \sin x)$, where c_1, c_2 are arbitrary constants. **[05]**

(B) Solve : **(Any Three)** **[12]**

(a) $(1 + y^2) + \left(x - e^{-\tan^{-1} x}\right) \dfrac{dy}{dx} = 0.$

(b) $\left(y^2 e^{xy^2} + 4x^3\right) dx + \left(2xye^{xy^2} - 3y^2\right) dy = 0.$

(c) $(x^2y + y^4)\, dx + (2x^3 + 4xy^3)\, dy = 0.$

(d) $\cos x \dfrac{dy}{dx} + y \sin x = \sqrt{y \sec x}\,.$

Q.3) Attempt **any three** of the following :

(a) The temperature of water initially is 100°C and that of surrounding is 20°C. If the water cools down to 60°C in first 20 minutes, what will be the time required to fall temperature up to 30°C ? **[05]**

(b) Form the differential equation for the circuit containing a resistance 'R' and a condensor of capacity 'C' in series with emf $E_0 \sin wt$. Find current at any instant t.

(Given $i = 0$ at $t = 0$) **[06]**

(c) For steady heat flow through the wall a hollow sphere of inner and outer radii r_1 and r_2 respectively, the temperature u at a distance r ($r_1 < r < r_2$) from the centre of sphere is given by

$$r \frac{d^2u}{dr^2} + 2\frac{du}{dr} = 0.$$

If u_1 and u_2 are the temperatures at inner and outer surfaces respectively. Find u in terms of r. **[06]**

(d) A bullet is fired into sand tank, its retardation is proportional to square root of its velocity. Show that the bullet will come to rest in time $\frac{2\sqrt{v}}{k}$, where v is initial velocity. **[05]**

OR

Q.4) Attempt **any three** of the following :

(a) Find orthogonal trajectories for the family of parabolas $y^2 = 4ax$. **[05]**

(b) A resistance of 100 ohms and an inductance of 0.5H are connected in series with a battery of 20 volts. Find the current in the circuit when initially $i = 0$ at $t = 0$. **[05]**

(c) A point executing S.H.M. has velocities v_1 and v_2 and acceleration a_1 and a_2 in two positions respectively. Show that distance between two positions is $\left|\frac{v_1^2 - v_2^2}{a_1 - a_2}\right|$. **[06]**

(d) In a chemical reaction in which two substances A and B initially of amounts a and b respectively are concerned. The velocity of transformation $\frac{dx}{dt}$ at any time t is known to be equal to the product "(a – x) (b – x)" of the amounts of the two substances then remaining untransformed. Find t in terms of x if a = 0.7, b = 0.5 and x = 0.3 when t = 300 seconds. **[06]**

Q.5) (A) Obtain Fourier series for

$$f(x) = \begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2 - x), & 1 \le x \le 2 \end{cases}$$ with period 2. **[07]**

(B) If $I_n = \int_0^{\pi/4} \frac{\sin(2n-1)x}{\sin x}\,dx$, then prove that

$I_{n+1} - I_n = \frac{1}{n}\sin\frac{n\pi}{2}$ and hence evaluate I_3. **[05]**

(C) Evaluate : $\int_0^{\infty} x^2 e^{-h^2x^2}\,dx$. **[04]**

OR

Q.6) (A) A turning moments y units of the crank of a steam engine is given for the series of values of crank angle θ in degrees :

θ	0	30	60	90	120	150	180
y	0	5224	8097	7850	5499	2626	0

Find first four moments in the series of sines to represent y. **[08]**

(B) Evaluate : $\int_0^{\pi} x\sin^7 x \cos^4 x\,dx$ **[04]**

(C) Prove that :

$$\int_0^1 x^{m-1}\left(1-x^2\right)^{n-1} dx = \frac{1}{2}\beta\left(\frac{m}{2}, n\right)$$ **[04]**

SECTION - II

Q.7) (A) Trace the following curves : **(Any Two)** **[08]**

(a) $y^2(2a-x) = x^3$

(b) $x^3 + y^3 = 3axy \ (a > 0)$

(c) $r = a\cos 3\theta$

(B) Find length of arc of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ intercepted in the positive quadrant. **[04]**

(C) Show that :

$$\int_0^{\infty} \frac{e^{-x} - e^{-ax}}{x.\sec x} dx = \frac{1}{2}\log\left(\frac{a^2+1}{2}\right).$$ **[05]**

OR

Q.8) (A) Trace the following curves : **(Any Two)** **[08]**

(a) $xy^2 = a(x^2 - a^2)$

(b) $x = a\cos^3 t,\ y = a\sin^3 t$

(c) $r^2 = a^2\cos 2\theta$.

(B) If $\alpha(x) = \sqrt{\frac{2}{\pi}} \int_0^x e^{-t^2/2}\, dt$, then show that $\text{erf}(x) = \alpha\left(x\sqrt{2}\right)$. **[04]**

(C) If $\phi(a) = \int_{\pi/6a}^{\pi/2a} \frac{\sin ax}{x} dx$, then find $\phi^1(a)$ and show that $\phi(a)$ is independent of a. **[05]**

Q.9) (A) Find equation of sphere which has its centre at (2, 3, –1) and touches line $\frac{x+1}{-5} = \frac{y-8}{3} = \frac{z-4}{4}$. **[05]**

(B) Find equation of cone whose vertex is at (1, 1, 3) and passes through guiding curve $4x^2 + z^2 = 1,\ y = 4$. **[05]**

(C) Find equation of right circular cylinder of radius 2, whose axis passes through (1, 2, 3) and has direction ratios proportional to (2, 1, 2). **[06]**

OR

Q.10) (A) Find equation of sphere which passes through the points (1, 0, 0); (0, 1, 0); (0, 0, 1) and having radius as small as possible. **[05]**

(B) Find equation of right circular cone with vertex at (1, –1, 1), semivertical angle is 45° and its axis is perpendicular to the plane $2x + y - 2z + 1 = 0$. **[06]**

(C) Find equation of cylinder whose guiding curve is $ax^2 + by^2 = 2z$, $lx + my + nz = p$ and generators are parallel to z-axis. **[05]**

Q.11) (A) Express the following integral as single integral and hence evaluate $\int_0^1 \int_0^y (x^2 + y^2)\, dxdy + \int_1^2 \int_0^{2-y} (x^2 + y^2)\, dxdy$. **[06]**

(B) Find area of the upper half of the cardiode $r = a(1 + \cos\theta)$. **[05]**

(C) Evaluate :

$$\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dz\, dx\, dy$$ **[06]**

OR

Q.12) (A) Find mean value of the function $e^{-(x^2 + y^2)}$ over the area of the circle $x^2 + y^2 = 1$. **[05]**

(B) Find the centroid of the area bounded by the curve $y^2(2a - x) = x^3$ and its asymtote. **[06]**

(C) Find the moment of inertia of a Lamina with uniform thickness bounded by $x^2 = y$ and $y = x + 2$ about y–axis. **[06]**
