S.E. (Computer Engg.) (Semester – I) Examination, 2010
DISCRETE STRUCTURES (Common to I.T.)
(2003 Course)

Time : 3 Hours Max. Marks : 100

Instructions : *1) Answer **any 3** questions from **each** Section.*

*2) Answers to the **two** Sections should be written in **separate** books.*

*3) Black figures to the **right** indicate **full** marks.*

SECTION – I

1. a) Prove by induction that the sum of the cubes of three consecutive integers is divisible by g. **8**

b) In the survey of 100 new cars, it is found that 60 had air conditioner (AC), 48 had power staring (PS), 44 had power-windows (PW), 36 had AC + PW, 20 had PS + AC, 16 had PW+PS, 12 had all three features. Find the number of cars that had. **8**

i) Only power window

ii) PS and PW but not AC

iii) AC and PS but not PW

Draw Venn diagram.

OR

2. a) Show the following using Venn diagram **5**

i) $A \cup (\overline{B} \cup C) = (A \cup \overline{B}) \cap (A \cap C)$

ii) $(A - B) - C = A - (B \cup C)$

b) Obtain the conjuctive normal form of each of the following : 6

i) $P \wedge (P \rightarrow Q)$

ii) $\sim(P \vee Q) \leftrightarrow (P \wedge Q)$

iii) $Q \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$.

c) Let P denote the statement 'The material is interesting'.

'q denote the statement 'The exercises are challenging' and r denote the statement', 'The course is enjoyable' write the following statements in symbolic form. 5

i) The material is interesting and exercises are challenging.

ii) The material is interesting means the exercises are challenging and conversely.

iii) Either the material is interesting or the exercises are not challenging but not both.

iv) If the material is not interesting and exercises are not challenging, then the course is not enjoyable.

v) The material is uninteresting, the exercises are not challenging and the course is not enjoyable.

3. a) i) In how many ways can the letter in the word MISSISSIPI be arranged ? 4

ii) In how many ways can three examinations be scheduled within a five day period so that no two examinations are scheduled on the same day ?

iii) Compute number of permutations of the set given

$\{1, 2, 3, 4, 5\}$.

iv) Suppose that repetitions are not permitted. How many four digit numbers can be formed from the six digits 1, 2, 3, 5, 7, 8 ?

b) Suppose that 3 balls are selected at random from an user containing 7 red balls and 5 black balls. Compute the probability that 6

i) All three balls are red

ii) At least two balls are black

iii) At most two balls are black

iv) At least one ball is red.

c) Define the terms with examples 6

i) Rule of sum and Rule of product

ii) Discrete probability and conditional probability

iii) Binominal distribution.

OR

4. a) A man has 7 relatives, 4 of them are ladies and 3 are gentleman, his wife has 7 relatives, 3 of them are ladies and 4 are gentlemen. In how many ways, can they invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of man's relatives and 3 of wife's relatives ? 6

b) Two cards are drawn at random from a deck of 52 cards. Find the probability p that 6

i) Both are diamonds

ii) One is diamond and one is heart.

c) A student has to answer 10 out of 13 questions in an examination : 4

i) How many choices he has ?

ii) How many choices he has if has to answer the first two questions ?

iii) How many choices he has if he must answer at least three out of first five ?

iv) How many choices he has if he must answer exactly three out of first five ?

5. a) Find the transitive clouser of R by Warshall's Algorithm where A = {1, 2, 3, 4, 5} and R = {(1, 1), (1, 2), (1, 3), (1, 4), (3, 1), (3, 2), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5)}.

Find the transitive closure for R using Warshall's Algorithm. 6

b) Three functions F, G, H are defined from $R \rightarrow R$ as follows :

i) $f(x) = 2x^3 + 5$

ii) $g(x) = \cos(x)$

iii) $h(x) = x^3 - 1$.

Find h_0 (gof) and (hog) of

Are they equal ? 4

c) Find the numeric function for 4

$$A(z)=\frac{2}{1-4z^2}$$

d) Define the following terms with example: 4

i) Antisymmetric relation

ii) Transitive relation

iii) Irreflexive relation.

OR

6. a) Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 18, 24} be ordered by the relation x divides y, show that the relation is partial ordering and draw the Hasse diagram. 5

b) Write short note on : chains and antichains. 4

c) What is recurrence relation ? Solve the following recurrence relation. 5

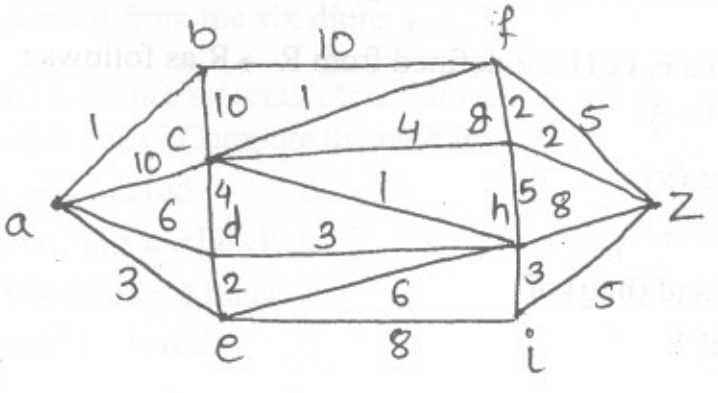
i) $a_r - 7\,a_{r-1} + 10\,a_{r-2} = 0$ given that $a_0 = 0$ and $a_1 = 3$.

ii) $a_r - 4\,a_{r-1} + 4\,a_{r-2} = 0$ given that $a_0 = 1$ and $a_1 = 6$.

d) Let X = {1, 2, ... 7} and R = {(x, y) / x–y, is divisible by 3}. Show that R is equivalence relation. Draw graph of R. 4

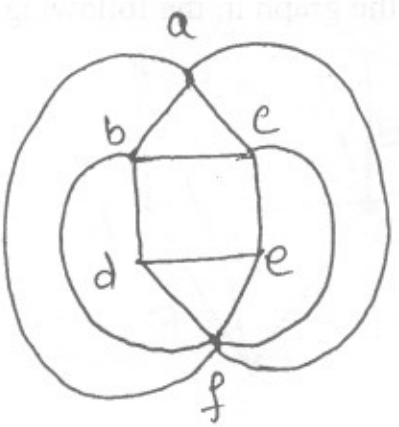
SECTION – II

7. a) Determine a shortest path between a and z in the graph, where the numbers associated with the edges are the distances between vertices. 8

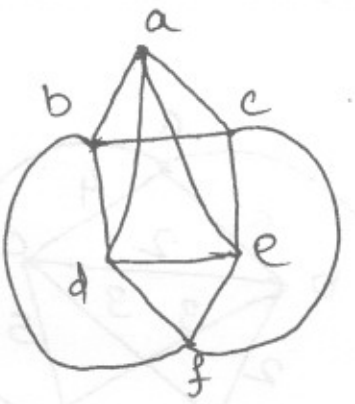


b) Define weighted graph, subgraph and factors of graph with a suitable example. **6**

c) For the following graphs determine whether the graph has an Euler's circuit and Euler's path.



(i)



(ii) **4**

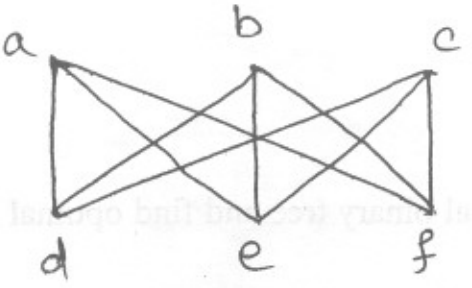
OR

8. a) Show that the graph G and G* are isomorphic G = (V, E) and G* = (V*, E*) given by,

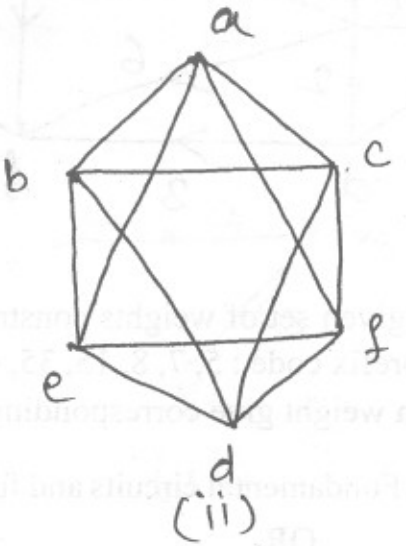
G = ({a, b, c, d}, {(a, b), (a, d), (b, d), (c, d), (c, b), (d, c)}).

G* = ({1, 2, 3, 4}, {(1, 2), (2, 3), (3, 1), (3, 4), (4, 3), (4, 2)}). **6**

b) Which of the following graphs represent bipartite graph and planar graph ? If planar graph redraw the same.



(i)



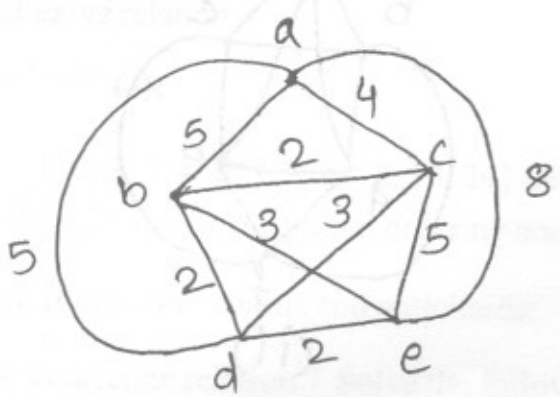
(ii) **6**

c) For the following figure solve the following options :

i) Use nearest neighbourhood method to find out Hamiltonian circuit for the graph starting at vertex 'a'.

ii) Repeat the part (i) starting at vertex 'd' instead.

iii) Determine the minimum Hamiltonian ckt for the graph in the following figure.



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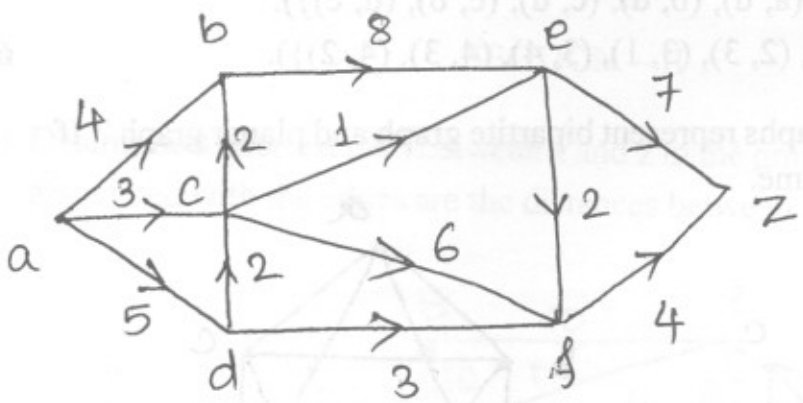
9. a) Use labelling procedure to find a maximum flow in the transport network shown in the diagram. Determine corresponding minimum cut.



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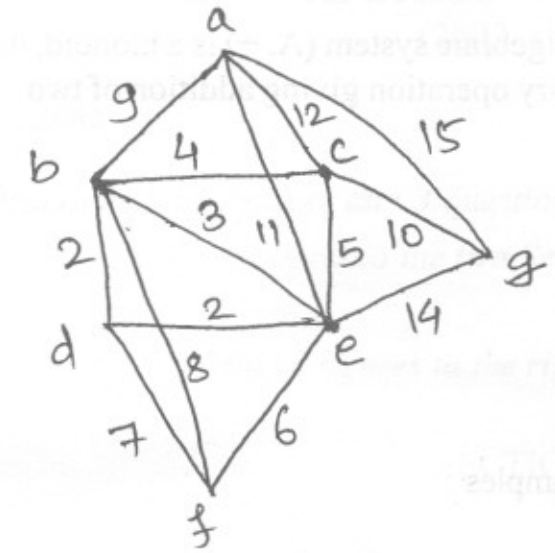
b) For the given set of weights construct optimal binary tree and find optimal binary prefix code : 5, 7, 8, 15, 35, 40.
For each weight give corresponding code word. 6

c) Explain Fundamental circuits and fundamental cut sets. 4

OR

10. a) Determine minimum spanning tree for the following graph using Kruskal's Algorithm.



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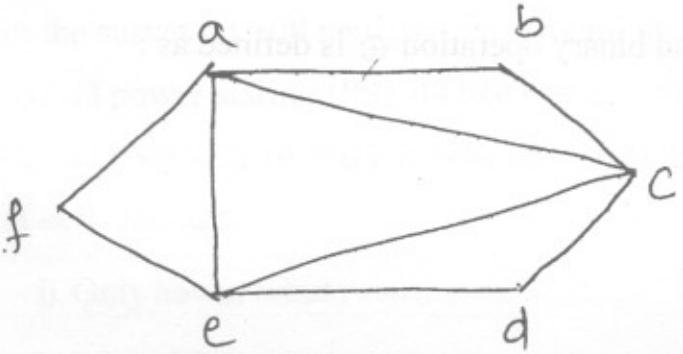
b) What do you understand by factors of a graph ? Find all possible k-factors of the following graph.



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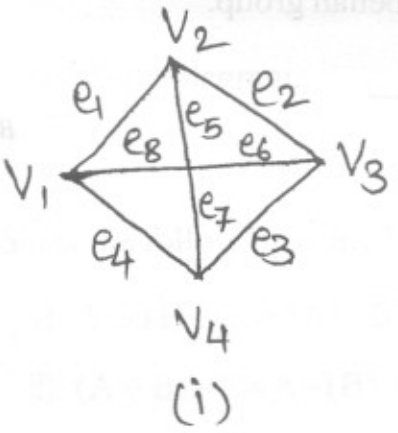
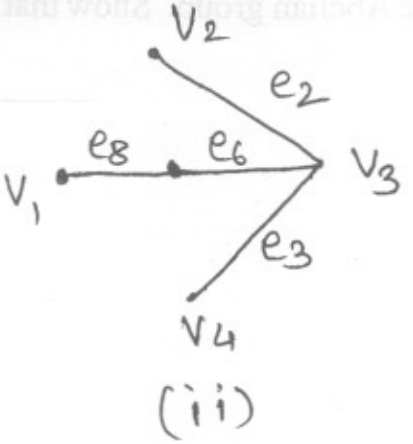
c) Find the fundamental system of circuits and the fundamental system of cut-sets for the following graph : i) With respect to the spanning tree given in (ii).





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11. a) Define Group. Show that $<z, *)$ is a group, where z is set of all integers that are divisible by 2 and * is a binary operation giving multiplication of 2 integers. **6**

b) What is monoid ? Show that the Algebraic system (A, +) is a monoid, where A is a set of integers and + is a binary operation giving addition of two integers. **6**

c) Define :

i) Ring Homomorphism

ii) Ring Isomorphism. **4**

OR

12. a) Define the following terms with examples

i) Ring

ii) Field

iii) Integral domain. **6**

b) Let G = {EVEN, ODD} and binary operation $\oplus$ is defined as :

$\oplus$	EVEN	ODD
EVEN	EVEN	ODD
ODD	ODD	EVEN

Show that $(G, \oplus)$ is a group. **6**

c) Define Abelian group. Show that $(Z_6, +)$ is Abelian group. **4**
