

S.E (I.T. & Comp.) (First Semester) EXAMINATION, 2010**DISCRETE STRUCTURES****(2008 Course)****Time : Three Hours****Maximum Marks : 100**

N.B. :— (i) Attempt from Section I Q. 1 or Q. 2, Q. 3 or Q. 4, Q. 5 or Q. 6. Attempt from Section II Q. 7 or Q. 8, Q. 9 or Q. 10, Q. 11 or Q. 12.

(ii) Answers to the two Sections should be written in separate answer-books.

(iii) Neat diagrams must be drawn wherever necessary.

(iv) Assume suitable data, if necessary.

SECTION I

1. (a) Prove by induction for $n \geq 0$ [6]

$$1 + a + a^2 + \dots + a^n = \frac{1 - a^{n+1}}{1 - a}.$$

(b) In a survey of 60 people it was found that : [6]

25 read Business India

26 read India Today

26 read Times of India

11 read both Business India and India Today

9 read both Business India and Times of India

8 read both India Today and Times of India

8 read none of these.

(i) How many read all three ?

(ii) How many read exactly one ?

(c) Prove that $[(p \rightarrow q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (q \vee s)$ is a tautology. [4]

Or

2. (a) Let P and Q be 2 multisets.

$P = \{a, a, a, c, d, d\}$ and $Q = \{a, a, b, c, c\}$. Find : [4]

(i) $P \cup Q$

(ii) $P \cap Q$

(iii) $P - Q$

(iv) $P + Q$

(b) $P(x) : x$ is even. [6]

$Q(x) : x$ is a prime number.

$R(x, y) : x + y$ is even.

(1) Using above write an English sentence for each of the symbolic statement given below :

(i) $\forall x (\sim Q(x))$

(ii) $\exists y (\sim P(y))$

(iii) $\sim (\exists x (P(x) \wedge Q(x)))$

(2) Using the information given above write the following English sentences in symbolic form :

(i) The sum of any two integers is an odd integer

(ii) Every integer is even or prime

(iii) Every integer is an odd integer.

(c) Find the CNF and DNF for the following : [4]

(i) $(p \rightarrow q) \wedge (q \rightarrow p)$

(ii) $((p \wedge (p \rightarrow q)) \rightarrow q)$

(d) Define power set.

List all elements of the set $p(A) \times A$ where $A = \{a, b, c\}$. [2]

3. (a) Show that $(I, \oplus, \odot)$ is a commutative ring with identity. Where $+$ and $\odot$ are defined as : [6]

$A \oplus b = a + b - 1$ and $a \odot b = a + b - ab$.

(b) Let Z_n denote the set of Integers as $\{1, \dots, n-1\}$.

Construct the multiplication table for $\odot$ with $n = 6$. Is

$(Z_n, \odot)$ [6]

Where $\odot$ is a binary operation on Z_n such that $a \odot b =$ remainder of ab divided by n . Is Z_n an abelian group ?

(c) Let G be a group of real nos under addition and G' be the group of +ve real nos under multiplication. Let $f : G \rightarrow G'$ be defined as $f(x) = e^x$. Show that f is an isomorphism. [4]

Or

4. (a) Define : [4]

(i) Subgroup

(ii) Cyclic Group

(iii) Integral domain

(iv) Field

(b) Prove the following results for the group G : [6]

(i) The identity element is unique.

(ii) Each a in G has a unique inverse a^{-1} .

(iii) $ab = ac$ implies $b = c$.

(c) Consider the (3, 6) encoding function e : [6]

$$e(001) = 000000$$

$$e(001) = 000110$$

$$e(010) = 010010$$

$$e(011) = 010100$$

$$e(100) = 100101$$

$$e(101) = 100011$$

$$e(110) = 110111$$

$$e(111) = 110001$$

Show that e is a group code.

5. (a) Let $A = B$ be the set of real nos. [6]

$f : A \rightarrow B$ given by $f(x) = 2x^3 - 1$

$$g : B \rightarrow A \text{ given by } g(y) = \sqrt[3]{\frac{1}{2}y + \frac{1}{2}}$$

Show that f is a bijection between A and B and g is bijection between B and A .

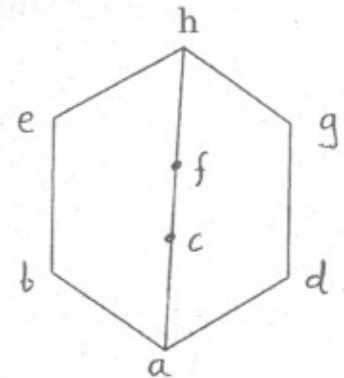
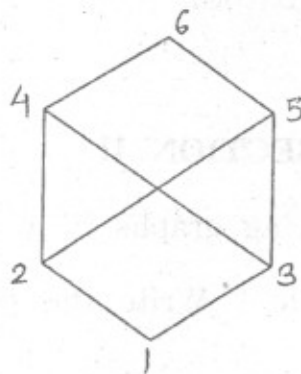
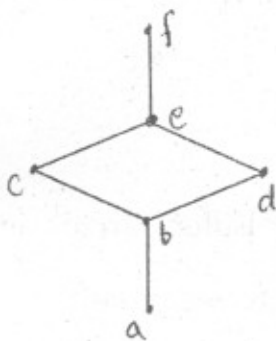
- (b) For each of these relations on set $A = \{1, 2, 3, 4\}$ decide whether it is reflexive, symmetric, transitive or antisymmetric. (one relation may satisfy more than one properties). [6]

$$R_1 = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$$

$$R_2 = \{(1, 1), (1, 2), (2, 2), (2, 1), (3, 3), (4, 4)\}$$

$$R_3 = \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)\}$$

- (c) Determine whether the poset represented by each of the Hasse diagram are lattices. Justify your answer. [6]



Or

6. (a) Find the solution to the recurrence relation

$$a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$$

with initial condition $a_0 = 2$, $a_1 = 5$ and $a_2 = 15$. [6]

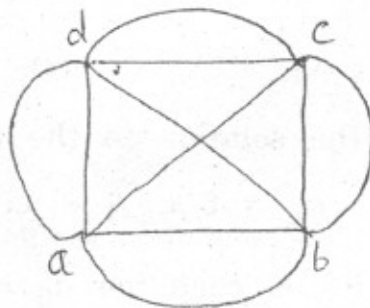
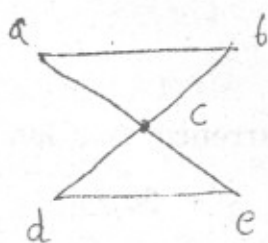
- (b) $A = \{1, 2, 3, 4, 5\}$ and R and S be equivalent relations on A whose matrices are given below. Compute the matrix of smallest relation containing R & S . [6]

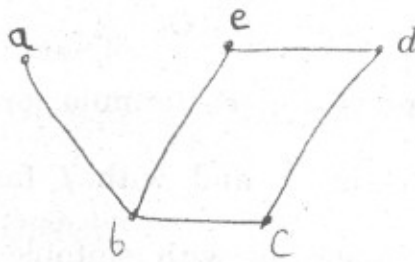
$$M_R = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \quad M_S = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- (c) Define with examples : [6]
- (i) Poset
 - (ii) Lattice
 - (iii) Complemented Lattice.

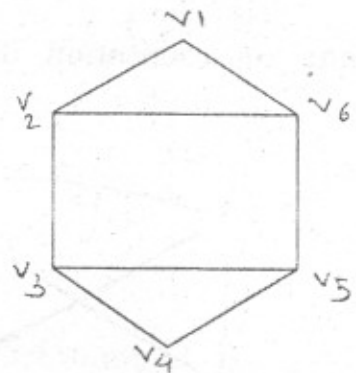
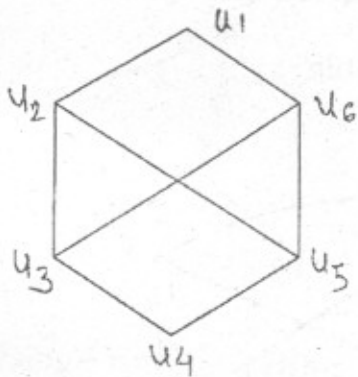
SECTION II

7. (a) Which of the following graphs have a Euler circuit or path or Hamiltonian cycle ? Write the path or circuit : [6]

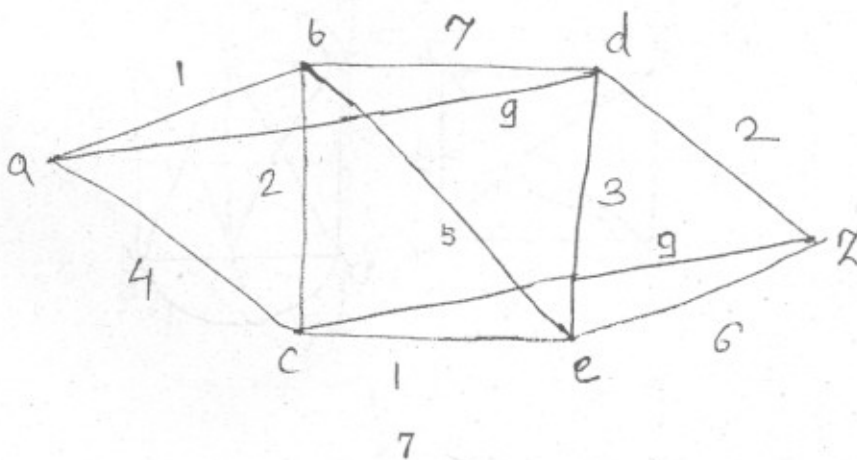




- (b) Determine whether graphs G and H are isomorphic or not. Justify your answer. [6]

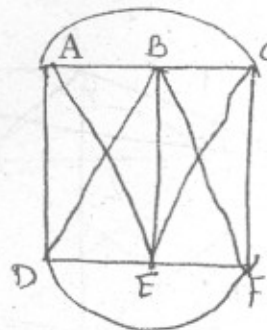
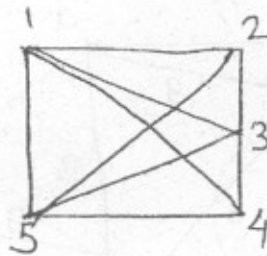
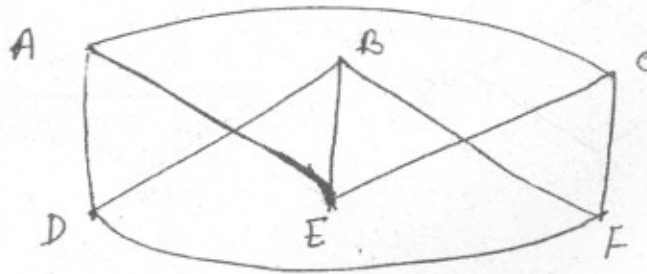


- (c) Find the shortest path from a to z in the following graph. [6]



Or

8. (a) State and prove Euler's formula for a connected planar graph of order n , size e and with f faces. [6]
- (b) Define the following with suitable example : [6]
- (i) Cut set
 - (ii) Factors of graph
 - (iii) Weighted graph.
- (c) Identify whether the graphs given are planar or not. Draw planar representation if possible : [6]

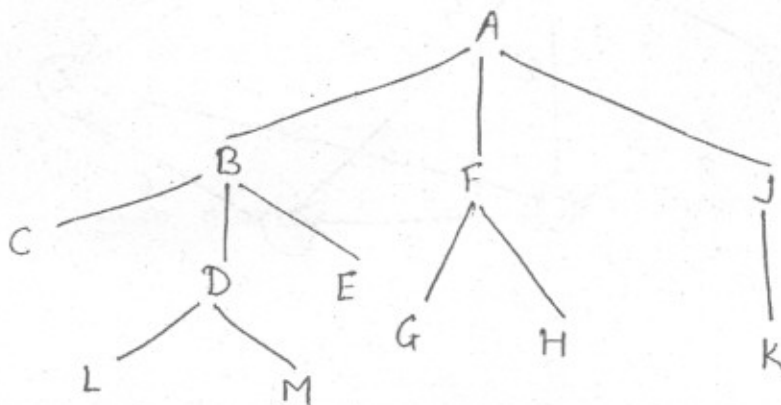


9. (a) A binary tree has 10 nodes. The inorder and preorder traversals of the trees are as shown below. Construct the binary tree. [6]

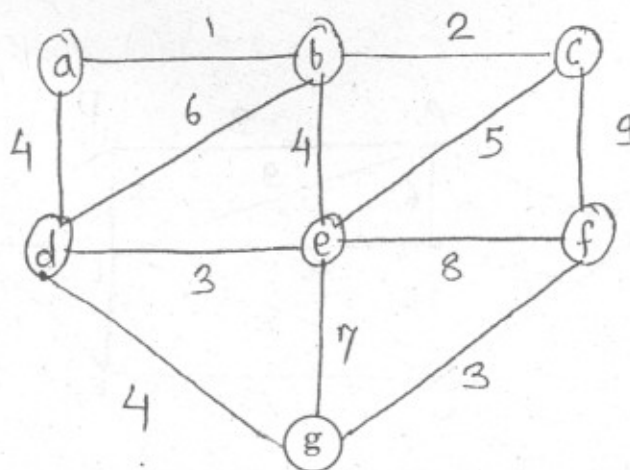
Inorder : ABCEDFJGIH

Preorder : JCBADEFIGH

- (b) Convert the following tree into binary tree. [4]

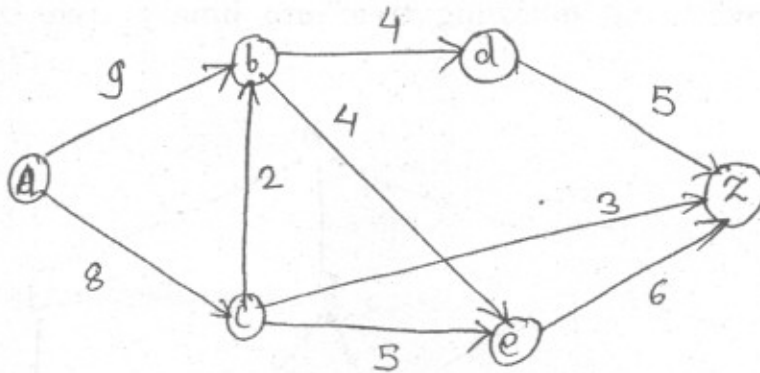


- (c) Using Prim's algorithm construct minimal spanning tree starting at vertex a. [6]

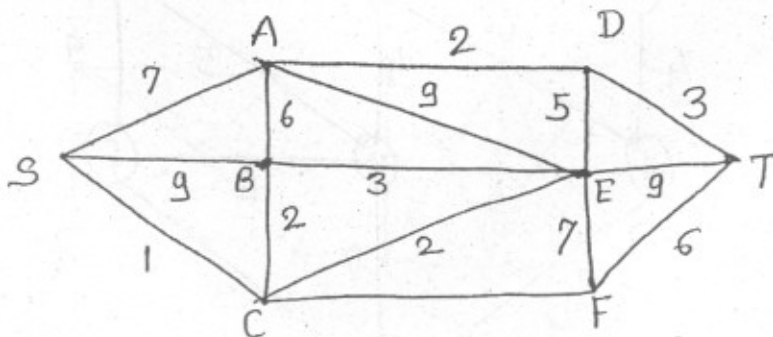


Or

10. (a) Find the maximum flow in the transport network given below : [6]



- (b) Construct the expression tree for the following expression.
 $(3 - (2(- 11 - (9 - 4)))) \div (2 + (3 + (4 + 7)))$. [4]
 Also evaluate the expression.
- (c) Using Kruskal's algorithm construct minimal spanning tree. [6]



11. (a) A single card is drawn from an ordinary deck of 52 cards. Find the probability p that : [3]
- (i) the card is a face card
 - (ii) the card is face card and heart
 - (iii) the card is face card or heart.
- (b) How many seven letter words can be formed using the letters of the word BENZENE ? [3]
- (c) Two dice are tossed once. Find the probability of getting an even number on first or a total of 8. [4]
- (d) If repetitions are not permitted, how many four digit numbers can be formed from digits 1, 2, 3, 7, 8, and 5. [6]

Or

12. (a) How many ways can the letters in the word MISSISSIPPI be arranged ? What if P's are to be separated ? [4]
- (b) Show that : [6]
- $$C(2n, 2) = 2C(n, 2) + n^2.$$
- (c) A pair of fair dice is thrown. Find the probability p that the sum is 10 or greater if : [3]
- (i) 5 appears on first die
 - (ii) 5 appears on at least one die.

(d) A coin is tossed 3 times. Find the probability that there will appear : [3]

(i) Three heads

(ii) Exactly 2 heads

(iii) No heads.