Total No. of Questions—**12**] [Total No. of Printed Pages—**8+2**

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S.E. (COMP)(Second Semester) EXAMINATION, 2010

(Common to Elect., Instru. & I.T.)

ENGINEERING MATHEMATICS—III

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

N.B. :— (*i*) In Section I, attempt Q. No. **1** *or* **2**, Q. No. **3** *or* **4**, Q. No. **5** *or* **6**. In Section II, attempt Q. No. **7** *or* **8**, Q. No. **9** *or* **10**, Q. No. **11** *or* **12**.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Figures to the right indicate full marks.

(*iv*) Neat diagrams must be drawn wherever necessary.

(*v*) Use of non-programmable electronic pocket calculator is allowed.

(*vi*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Solve any *three* : [12]

(*i*) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$

(*ii*) $\left(D^2 - 1\right) y = x \sin x + \left(1 + x^2\right) e^x$

(*iii*) $y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$ (By Variation of Parameters)

(*iv*) $\left(x^2D^2 - xD + 1\right) y = x \log x$

(*v*) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{1}{1+e^x}$.

(*b*) An uncharged condenser of capacity C charged by applying an e.m.f. of value $E \sin \frac{t}{\sqrt{LC}}$ through the leads of inductance L and of negligible resistance. The charge Q on the plate of condenser satisfies the differential equation :

$$\frac{d^2Q}{dt^2} + \frac{Q}{LC} = \frac{E}{L} \sin \frac{t}{\sqrt{LC}}.$$

Prove that the charge at any time t is given by

$$Q = \frac{EC}{2}\left[\sin \frac{t}{\sqrt{LC}} - \frac{t}{\sqrt{LC}} \cos \frac{t}{\sqrt{LC}}\right].$$ [5]

Or

2. (*a*) Solve any *three* : [12]

(*i*) $\frac{d^3y}{dx^3} + 4\frac{dy}{dx} = \sin 2x$

(*ii*) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$

(*iii*) $\dfrac{d^2y}{dx^2} + y = \tan x$ (By Variation of Parameters)

(*iv*) $\dfrac{dx}{x\left(2y^4 - z^4\right)} = \dfrac{dy}{y\left(z^4 - 2x^4\right)} = \dfrac{dz}{z\left(x^4 - y^4\right)}$

(*v*) $\left(D^4 - 2D^3 - 3D^2 + 4D + 4\right) y = x^2e^x.$

(*b*) Solve : [5]

$$\frac{dx}{dt} + 5x - 2y = t$$

$$\frac{dy}{dt} + 2x - y = 0.$$

3. (*a*) If

$$u = \frac{1}{2}\log\left(x^2 + y^2\right),$$

find v such that $f(z) = u + iv$ is analytic. Determine $f(z)$ in terms of z. [5]

(*b*) Evaluate :

$$\oint_C \frac{z^2 + 1}{z - 2}\, dz$$

where

(*i*) C is the circle $|z - 2| = 1$

(*ii*) C is the circle $|z| = 1.$ [5]

(c) Find the bilinear transformation which maps the points $z = 1, i, 2i$ on the points $w = -2i, 0, 1$ respectively. [6]

Or

4. (a) If $f(z)$ is analytic, show that :

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^4 = 16|f(z)|^2 + |f'(z)|^2. \quad [5]$$

(b) Evaluate using residue theorem,

$$\int_C \frac{2z^2 + 2z + 1}{(z+1)^3 (z-3)} dz,$$

where C is the contour $|z + 1| = 2$. [6]

(c) Show that under the transformation,

$$w = \frac{i - z}{i + z},$$

x-axis in z-plane is mapped onto the circle $|w| = 1$. [5]

5. (a) Find the Fourier transform of :

$$f(x) = 1 - x^2, \quad |x| \leq 1$$
$$= 0, \quad |x| > 1$$

Hence evaluate :

$$\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right)\cos\frac{x}{2}\, dx. \quad [6]$$

(b) Prove that the Sine Fourier transform of :

$$f(x) = \frac{1}{x} \text{ is } \sqrt{\frac{\pi}{2}}.$$ [5]

(c) Find z-transform of the following (any *two*) : [6]

(i) $f(k) = 3^k \quad , \; k < 0$

$\quad\quad = 2^k \quad , \; k \geq 0$

(ii) $f(k) = \dfrac{\sin ak}{k} \quad , \; k > 0$

(iii) $f(k) = ke^{-ak} \quad , \; k \geq 0.$

Or

6. (a) Find inverse z-transform (any *two*) : [6]

(i) $F(z) = \dfrac{z}{\left(z - \frac{1}{4}\right)\left(z - \frac{1}{5}\right)}, \; |z| > \dfrac{1}{4}$

(ii) $F(z) = \dfrac{10z}{(z-1)(z-2)}$, By Inversion Integral Method

(iii) $F(z) = \dfrac{1}{(z-2)(z-3)}, \; |z| < 2$

(b) Solve the difference equation,

$$f(k+1) + \frac{1}{2}f(k) = \left(\frac{1}{2}\right)^k, \; k \geq 0, \; f(0) = 0.$$ [5]

(c) Solve the integral equation : [6]

$$\int_0^{\infty} f(x) \sin \lambda x \, dx = \begin{cases} 1 , & 0 \le \lambda < 1 \\ 2 , & 1 \le \lambda < 2 \\ 0 , & \lambda \ge 2 \end{cases}$$

SECTION II

7. (a) The first four moments about the working mean 3.5 of a distribution are 0.0375, 0.4546, 0.0609 and 0.5074. Calculate the moments about the mean. Also calculate the coefficients of skewness and kurtosis. [8]

(b) Calculate the coefficient of correlation between the marks obtained by 8 students in Mathematics and Statistics from the following table. Also find the lines of regression : [9]

Student	**Maths (x)**	**Statistics (y)**
A	25	8
B	30	10
C	32	15
D	35	17
E	37	20
F	40	22
G	42	24
H	45	25

Or

8. (*a*) 20% of bolts produced by a machine are defective. Determine the probability that out of 4 bolts chosen at random :

(*i*) 1 is defective

(*ii*) at most 2 bolts are defective. [6]

(*b*) A telephone switch board handles 600 calls on an average during rush hour. The board can make a maximum of 20 calls per minute. Use Poisson's distribution to estimate the probability, the board will be over taxed during any given minute. [5]

(*c*) In a distribution exactly normal, 7% of the items are under 35 and 89% are under 63. Find the mean and standard deviation of the distribution, using the following data.

(Normal variate corresponding to 0.43 is 1.48 and corresponding to 0.39 is 1.23.) [6]

9. (*a*) Find the constant '*a*' such that the tangent plane to the surface $x^3 - 2xy + yz = (a + 4)$ at the point $(2, 1, a)$ will pass through origin. [6]

(*b*) If $\bar{a}, \bar{b}$ are constant vectors and $\bar{r}$ and r have their usual meaning, then show that : [6]

(*i*) $\bar{a} \cdot \nabla \left(\bar{b} \cdot \nabla \frac{1}{r} \right) = \frac{3(\bar{a} \cdot \bar{r})(\bar{b} \cdot \bar{r})}{r^5} - \frac{\bar{a} \cdot \bar{b}}{r^3}$

(*ii*) $\nabla \times \left(\bar{a} \times \nabla \frac{1}{r} \right) + \nabla \left(\bar{a} \cdot \nabla \frac{1}{r} \right) = 0.$

(*c*) Show that :

$$\frac{d}{dt}\left[\bar{r} \cdot \left(\frac{d\bar{r}}{dt} \times \frac{d^2\bar{r}}{dt^2} \right) \right] = \bar{r} \cdot \left(\frac{d\bar{r}}{dt} \times \frac{d^3\bar{r}}{dt^3} \right). \quad [4]$$

Or

10. (*a*) If $\bar{a}$ is a constant vector and

$$\bar{F} = r\bar{a} + \left(\frac{\bar{a} \cdot \bar{r}}{r} \right) \bar{r},$$

then show that $\bar{F}$ is irrotational and hence find scalar potential ϕ such that $\bar{F} = \nabla\phi$. [6]

(*b*) Find the angle between the surfaces $xy^2 + z^3 + 3 = 0$ and $x \log z - y^2 + 4 = 0$ at $(-1, 2, 1)$. [4]

(c) If $\bar{r}_1$ and $\bar{r}_2$ are vectors joining the fixed points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ to the variable point $P(x, y, z)$, then show that :

(i) $\nabla\left(\bar{r}_1 \cdot \bar{r}_2\right) = \bar{r}_1 + \bar{r}_2$

(ii) $\nabla \times \left(\bar{r}_1 \times \bar{r}_2\right) = 2\left(\bar{r}_1 - \bar{r}_2\right)$. [6]

11. (a) Evaluate :

$$\int_C \bar{F} \cdot d\bar{r},$$

where $\bar{F} = 3y\,\hat{i} + 2x\,\hat{j}$ and 'C' is the boundary of a rectangle $0 \le x \le \pi;\ 0 \le y \le \sin x$. [5]

(b) Evaluate :

$$\iint_S \bar{F} \cdot d\bar{S},$$

where $\bar{F} = yz\,\hat{i} + xz\,\hat{j} + xy\,\hat{k}$, and 'S' is the surface of the sphere $x^2 + y^2 + z^2 = 1$, in the positive octant. [5]

(c) Verify Stokes' Theorem, for $\bar{F} = xy\,\hat{i} + xy^2\,\hat{j}$ and C is the square in XY-plane with vertices (1, 0), (–1, 0), (1, 1) and (–1, 1). [7]

Or

12. (*a*) Evaluate :

$$\int_C (\sin z\, dx - \cos x\, dy + \sin y\, dz),$$

where 'C' is boundary of the rectangle $0 \leq x \leq \pi$; $0 \leq y \leq 1$, $z = 3$. [5]

(*b*) Evaluate :

$$\iint_S \frac{dS}{\sqrt{a^2x^2 + b^2y^2 + c^2z^2}},$$

over the closed surface of the ellipsoid

$$ax^2 + by^2 + cz^2 = 1.$$ [7]

(*c*) If $\overline{F} = \nabla r^2$, and 'S' is any closed surface containing volume 'V', then show that :

$$\iint_S \overline{F} \,.\, d\overline{S} = 6V.$$ [5]