S.E. (Common to Comp./E.TC./I.T./Electrical S/W/Instru.) Examination, 2010
ENGINEERING MATHEMATICS – III (2003 Course)

Time : 3 Hours Max. Marks : 100

Instructions : *1) Answers to the **two** Sections should be written in **separate** books.*
*2) **Neat** diagrams must be drawn **wherever** necessary.*
*3) Black figures to the **right** indicate **full** marks.*
*4) **Use** of electronic pocket calculator is **allowed**.*
*5) Assume **suitable** data, if **necessary**.*

SECTION – I

1. a) Solve the following (**any three**) : **12**

i) $(D-1)^3 y = e^{3x} + 3^x + \frac{7}{2}$

ii) $\frac{d^2y}{dx^2} + y = x\sin x$ [By variation of parameter]

iii) $(D^2 - 4D + 4)y = e^{2x}\sin 3x$

iv) $(D^2 - 4D + 3)y = x^3 e^{2x}$

v) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \sin[\log x^2]$

b) A circuit consists of an inductance L and condenser of capacity C is in series. An alternating e.m.f. Esinnt is applied to it at time t = 0, initial current and the charge on the condenser being zero. Find the current flowing in the circuit at any time for w = n. **5**

OR

2. a) Solve the following (**any three**) : **12**

i) $\frac{d^3y}{dx^3} - 4\frac{dy}{dx} = \sinh 2x$

ii) $(D^2 + 1)y = \cos 2x \cdot \cos x$

iii) $(D^2 - 6D + 9)y = \frac{e^{3x}}{x^2}$ [By variation of parameter]

iv) $(D^2 - 2D + 5)y = 25x^2$

v) $(2x+1)^2\frac{d^2y}{dx^2} - 2(2x+1)\frac{dy}{dx} - 12y = 6x$

b) Solve the simultaneous equations, 5

$$\frac{dx}{dt} - wy = a\cos pt$$

$$\frac{dy}{dt} + wx = a\sin pt$$

3. a) If $v = \frac{-y}{x^2 + y^2}$ find u such that $f(z) = u + iv$ is analytic. Determine $f(z)$ in terms of z. 6

b) Evaluate $\oint_c \frac{z+2}{z^2+1} dz$ where $C : |z - i| = \frac{1}{2}$. 5

c) Find the Bilinear transformation which sends the points 1, i, –1 from z-plane into the points i, o, –i of the w-plane. 5

OR

4. a) Evaluate $\int_c \frac{\cos \pi z^2}{(z-1)(z-2)} dz$ where C is the circle $|z| = 3$. 6

b) Show that, under the transformation $w = \frac{i - z}{i + z}$, x-axis in z-plane is mapped onto the circle $|w| = 1$. 5

c) Show that the function $f(z)$ with constant magnitude is constant. 5

5. a) Find Fourier sine and Fourier cosine transform of $f(x) = \begin{cases} \sin x & 0 \le x < a \\ 0 & x > a \end{cases}$. 6

b) Find the inverse Fourier sine transform of $Fs(\lambda) = \frac{e^{-a\lambda}}{\lambda}$. 6

c) Find inverse z-transform of $\frac{z+2}{z^2 - 2z + 1}$ $|z| > 1$. 5

OR

6. a) Find z-transform of the following (**any two**) :

i) $f(k) = \frac{2^k}{k!}$ $k \ge 0$

ii) $f(k) = (1 + k)a^k$ $k \ge 0$

iii) $f(k) = 2^k \sinh \alpha k$ $k \ge 0$ 6

b) Using Fourier Integral representation show that

$$e^{-x} - e^{-2x} = \frac{6}{\pi}\int_0^\infty \frac{\lambda \sin \lambda x}{(\lambda^2+1)(\lambda^2+4)} d\lambda, x > 0.$$ 6

c) Find the Fourier cosine transform of $f(x) = \begin{cases} x & 0 \le x \le 1/2 \\ 1-x & 1/2 < x < 1 \\ 0 & x > 1 \end{cases}$ 5

SECTION – II

7. a) Find the Laplace transforms of **any two** of the following : 8

i) $4t + e^{-2t}t^2 + t \sin t$

ii) $\int_0^t \left[\frac{e^t - \cos 2t}{t}\right] dt$

iii) $\frac{\cos mt - \cos nt}{t}$

b) Evaluate the integral by using Laplace transform $\int_0^\infty e^{-3t} t \cos t \, dt$. 4

c) Solve the differential equation by using Laplace transform method $\frac{d^2y}{dt^2} + y = 0$ where $y(0) = 1,\ y'(0) = 2$. 4

OR

8. a) Find Inverse Laplace transforms of any two of the following : 8

i) $\frac{2s+3}{s^2+8s+2s}$

ii) $\frac{3s+1}{(s-4)(s+3)}$

iii) $\frac{4s+3}{2s+1}$

b) Use convolution theorem to find Inverse Laplace transform of $\frac{1}{s(s^2+4)}$. 4

c) Obtain Laplace transform of $t^4 v(t-2) + t^3\,\delta(t-4)$. 4

9. a) Prove the following **(any two)** : 8

i) $\nabla.\left(r\nabla\frac{1}{r^n}\right) = \frac{n(n-2)}{r^{n+1}}$

ii) $\nabla\left(\nabla.\frac{\bar{r}}{r}\right) = -\frac{2\bar{r}}{r^3}$

iii) $\nabla^2 f(r) = f''(r) + \frac{2}{r}f'(r)$

b) Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at $(2, -1, 2)$ in the direction of $2i - 3j + 6k$. 5

c) Show that the vector field $[f(r)\bar{r}]$ is irrotational vector field. 4

OR

10. a) Show that the vector field $\overline{F} = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$ is conservative and hence find a scalar potential ϕ such that $\overline{F} = \nabla\phi$. 7

b) Show that curl[grad ϕ] = 0 where ϕ is some scalar point function. 5

c) If ϕ, ψ satisfies the Laplaces equation then prove that $(\phi\nabla\psi - \psi\nabla\phi)$ is solenoidal function. 5

11. a) Evaluate $\int_C yzdx + (xz+1)dy + (xy)dz$ where C is the curve from (0, 0, 0) to (2, 1, 4). 5

b) Verify the Green's Lemma for the vector field $\overline{F} = x^2 i + xy\,j$ over the region bounded between $y = x^2$ and $y = x$. 5

c) Verify Gauss divergence theorem for $\overline{F} = 4xzi - y^2 j + yzk$ over the surface s of the cube bounded between x = 0, x = 2, y = 0, y = 2, z = 0, z = 2. 7

OR

12. a) Verify Stokes theorem for $\overline{F} = (y - z + 2)i + (yz + 4)j - xzk$ over the surface of a cube x = 0, x = 2, y = 0, y = 2 and z = 2 above the XOY plane (open at the bottom). 7

b) Maxwell's equations are given by $\nabla \cdot \overline{H} = 0$, $\nabla \times \overline{E} = -\frac{\partial \overline{H}}{\partial t}$, $\nabla \times \overline{H} = \frac{\partial \overline{E}}{\partial t}$. Show that $\overline{H}$ satisfies the equation $\nabla^2 u = \frac{\partial^2 u}{\partial t^2}$. 5

c) Evaluate $\int_C \overline{F} \cdot d\overline{r}$ where $\overline{F} = 3xi + (2xz - y)j + zk$ from (0, 0, 0) to (2, 1, 3) along a line joining these points. 5

———————