Total No. of Questions—**12**] [Total No. of Printed Pages—**8+2**

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S.E. (Mech., Production, S/W)(First Sem.) EXAMINATION, 2010

ENGINEERING MATHEMATICS—III

(2008 COURSE)

Time : Three Hours **Maximum Marks : 100**

N.B. :— (*i*) Answer Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6** from Section I and Q. No. **7** *or* Q. No. **8**, Q. No. **9** *or* Q. No. **10**, Q. No. **11** *or* Q. No. **12** from Section II.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Figures to the right indicate full marks.

(*v*) Use of electronic pocket calculator is allowed.

(*vi*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Solve the following differential equations (any *three*) : [12]

(1) $\left(D^3 - D^2 - 6D\right)y = 1 + x^2$

(2) $\left(D^2 - 5D + 6\right)y = x \cos 2x$

(3) $x^2 \frac{d^2y}{dx^2} + 5x \frac{dy}{dx} + 3y = \frac{\log x}{x^2}$

(4) $\left(D^3 - 4D\right) y = 2 \cosh^2 (2x)$

(5) $\frac{x\, dx}{z^2 - 2yz - y^2} = \frac{dy}{y + z} = \frac{dz}{y - z}$.

(*b*) Solve the simultaneous differential equations :

$$4 \frac{dx}{dt} + x - y = 0, \; x + 2 \frac{dy}{dt} - y = 0$$

given $x = 20$ and $y = 100$ at $t = 0$. [5]

Or

2. (*a*) Solve the following differential equations (any *three*) : [12]

(1) $\left(D^2 + 6D + 9\right) y = 5^x - \log 2$

(2) $(D - 1)^2 \left(D^2 + 1\right) y = e^x + \sin^2 \frac{x}{2}$

(3) $(x + 1)^2 \frac{d^2y}{dx^2} + (x + 1) \frac{dy}{dx} = (2x + 3)(2x + 4)$

(4) $\left(D^2 - 1\right) y = \left(1 + e^{-x}\right)^2$

(5) $\left(D^2 - 2D + 1\right) y = x^{3/2} e^x$.

(by using variation of parameters method)

(b) A body weighing 20 kg is hung from a spring. A pull of 40 kg weight will stretch the spring to 10 cm. The body is pulled down to 20 cm below the state of equilibrium position and then released. Find the displacement of the body from its equilibrium position at time t secs. Also find maximum velocity and period of oscillation. [5]

3. (a) Find Laplace Transform of (any *two*) : [6]

(1) $e^t\left(1+\sqrt{t}\right)^3$

(2) $t\sqrt{1+\sin t}$

(3) $e^{-t}\sin t\, u(t-\pi)$.

(b) Solve using Laplace Transform method :

$$y'' + 4y' + 8y = 1$$

given $y(0) = 0,\ y'(0) = 1$. [5]

(c) Find Fourier transform of :

$$f(x) = \sin x \qquad 0 < x < \pi$$
$$= 0 \qquad x > \pi \text{ and } x < 0.$$ [6]

Or

4. (a) Find inverse Laplace Transform of (any *two*) : [8]

(1) $\dfrac{s+2}{s^2(s+3)}$

(2) $\dfrac{1}{(s-2)^4\,(s+3)}$ by convolution thm.

(3) $\log \dfrac{s^2+1}{s(s+1)}$.

(*b*) Evaluate :

$$\int_0^\infty e^{-2t}\, t^2 \sin 3t\, dt. \qquad [4]$$

(*c*) Solve the integral equation : [5]

$$\begin{aligned}\int_0^\infty f(x)\sin\lambda x\, dx &= 1 \quad 0 \le \lambda < 1\\ &= 2 \quad 1 \le \lambda < 2\\ &= 0 \quad \lambda > 2.\end{aligned}$$

5. (*a*) A homogeneous rod of conducting material of length 100 cm has its ends kept at zero temperature and the temperature initially is :

$$\begin{aligned}u(x,\ 0) &= x \qquad\qquad 0 \le x \le 50\\ &= 100 - x \qquad 50 \le x \le 100.\end{aligned}$$

Find the temperature $u(x,\ t)$ at any time. [8]

(*b*) The vibrations of an elastic string is governed by the partial differential equations :

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}.$$

The length of the string is π and the ends are fixed. The initial velocity is zero and the initial deflection is $u(x, 0) = 2(\sin x + \sin 3x)$. Find the deflection of the string for $t > 0$. [8]

Or

6. (*a*) Solve :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,$$

subject to the conditions :

(1) $u(0, y) = 0$

(2) $u(10, y) = 0$

(3) $u(x, \infty) = 0$

(4) $u(x, 0) = 20x \qquad 0 \le x \le 5$

$\qquad = 20(10 - x) \qquad 5 \le x \le 10.$ [8]

(*b*) Use Fourier sine transform to solve the equation :

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad 0 < x < \infty, \ t > 0$$

subject to the conditions :

(1) $u(0, t) = 0$

(2) $u(x, 0) = e^{-x} \quad x > 0$

(3) u & $\frac{\partial u}{\partial x} \to 0$ as $x \to \infty$. [8]

SECTION II

7. (*a*) Ten students got the following percentage of marks in Economics and Statistics :

Marks in Economics	**Marks in Statistics**
78	84
36	51
98	91
25	60
75	68
82	62
90	86
62	58
65	53
39	47

Calculate coefficient of correlation. [6]

(*b*) The probability that a bomb dropped from a plane will strike the target is $\frac{1}{5}$. If six bombs are dropped, find the probability that exactly two will strike the target. [5]

(*c*) Calculate the first four moments of the following distribution about the mean and hence find β_1 and β_2 : [6]

x	f
0	1
1	8

2	28
3	56
4	70
5	56
6	28
7	8
8	1

Or

8. (*a*) Goals scored by two teams A and B in a football season were as follows :

No. of Goals Scored in a Match	No. of Matches A	No. of Matches B
0	27	17
1	09	09
2	08	06
3	05	05
4	04	03

Find out which team is more consistent. [6]

(*b*) Between the hours 2 p.m. and 4 p.m. the average number of phone calls per minute into switch board of a company is 2.35. Find the probability that during one particular minute there will be at most 2 phone calls. [6]

(c) In a test on 2000 electric bulbs it was found that the life of a particular make was normally distributed with an average time 2040 hours and S.D. of 60 hours. Estimate the number of bulbs likely burn for more than 1920 hours but less than 2160 hours.

(Given $z = 2$, Area $= .4772$) [5]

9. (a) The acceleration of a particle at any time $t \geq 0$ is given by :

$$12\cos 2ti - 8\sin 2tj + 16tk.$$

The velocity and displacement are zero at $t = 0$. Find velocity and displacement at any time t. [6]

(b) If $(xyz)^b \left(x^a i + y^a j + z^a k\right)$ is an irrotational vector field, prove that either $b = 0$ or $a = -1$. [6]

(c) Find the directional derivative of $\text{div}\left(x^5 i + y^5 j + z^5 k\right)$ at (2, 2, 1) in the direction of outward normal to the surface $x^2 + y^2 + z^2 = 9$ at the point (2, 2, 1). [5]

Or

10. (a) Prove that (any *two*) : [6]

(1) $\phi = \dfrac{1}{r}$ satisfies Laplace equation

(2) $\nabla \times (\bar{a} \times \nabla \log r) = \dfrac{2(\bar{a} \cdot \bar{r})\bar{r}}{r^4}$

(3) $\nabla \cdot \left(r \nabla \dfrac{1}{r^3}\right) = \dfrac{3}{r^4}$.

(b) Show that the vector field $f(r)\bar{r}$ is always irrotational and determine $f(r)$ such that the field is solenoidal also. [6]

(c) If ϕ has at the point (1, 2) directional derivative +2 in the direction towards (2, 2) and −2 in the direction towards (1, 1). Find grad f at (1, 2). [5]

11. (a) Evaluate

$$\int_c \bar{F} \cdot d\bar{r}$$

where

$$\bar{F} = \left(3x^2 - 6yz\right)i + (2y + 3xz)\,j + \left(1 - 4xyz^2\right)k$$

along the line joining the points (0, 0, 0), (1, 2, 3). [5]

(b) Evaluate :

$$\int_s \bar{F} \cdot d\bar{s}$$

where

$$\bar{F} = 4xi - 2y^2 j + z^2 k$$

and s is surface bounding region $x^2 + y^2 = 4$, $z = 0$, $z = 3$. [6]

(c) Apply Stokes' theorem to evaluate :

$$\int_c y\,dx + z\,dy + x\,dz$$

where c is the curve of intersection of $x^2 + y^2 + z^2 = a^2$ and $x + z = a$. [5]

Or

12. (*a*) Use divergence theorem to evaluate :

$$\iint_s \left(2xyi + yz^2 j + xzk\right) . d\bar{s}$$

where s is surface of the region bounded by $x = 0$, $y = 0$, $z = 0$, $y = 3$, $x + 2z = 6$. [5]

(*b*) Verify Stokes' theorem in the plane $z = 0$ for $\bar{F} = \left(x - y^2\right)i + 2xyj$ for the region bounded by $y = 0$, $x = 2$, $y = x$. [6]

(*c*) Find the work done by :

$$\bar{F} = 2xy^2 i + \left(2x^2 y + y\right)j$$

in taking a particle from (0, 0, 0) to (2, 4, 0) along the parabola $y = x^2$, $z = 0$. [5]