S.E. (Mechanical) (Semester – I) Examination, 2010
ENGINEERING MATHEMATICS – III
(2003 Course)
Common to Mech. S/W, Prod. and Prod. S/W, Ind. Engg. Metallurgy Engg. (Sem. – II)

Time : 3 Hours Max. Marks : 100

Instructions : *i) Answers to the* ***two*** *Sections should be written in* ***separate*** *answer books.*

ii) In Section – ***I****, attempt Q. No.* ***1*** *or Q. No.* ***2****, Q. NO.* ***3*** *or Q. No.* ***4****, Q. No.* ***5*** *or Q. No.* ***6****.*

iii) In Section – ***II****, attempt Q. No.* ***7*** *or Q. No.* ***8****, Q. No.* ***9*** *or Q. No.* ***10****, Q. No.* ***11*** *or Q. No.* ***12****.*

iv) ***Neat*** *diagrams must be drawn* ***wherever*** *necessary.*

v) Figures to the ***right*** *indicate* ***full*** *marks.*

vi) ***Use*** *of non-programmable electronic pocket calculator is* ***allowed****.*

vii) Assume suitable data, ***if necessary****.*

SECTION – I

1\. a) Solve **any three** of the following : **12**

i) $\frac{d^2y}{dx^2}+\frac{dy}{dx}=\frac{1}{1+e^x}$

ii) $(D^2-4D+3)y=x^3e^{2x}$

iii) $(D^2+D+1)\,y = x\sin x$

iv) $\frac{d^2y}{dx^2}+y=\sec x\tan x$ (Use method of variation of parameters).

v) $x^2\frac{d^3y}{dx^3}+3x\frac{d^2y}{dx^2}+\frac{dy}{dx}+\frac{y}{x}=\log x.$

b) Solve $\frac{dx}{dt}+x-y=e^t,\ 2y-\frac{dx}{dt}+\frac{dy}{dt}=e^t.$ **5**

OR

2. a) Solve **any three** of the following : 12

i) $(D^2 + 5D + 6)\,y = e^{ex}$

ii) $(D^5 - D)\,y = 12e^x + 8\sin x - 2x$

iii) $(D^4 + 6D^3 + 13D^2 + 12D + 4)\,y = x^2e^x$

iv) $(D^2 - 2D + 2)y = e^x \tan x$ (Use method of variation of parameters)

v) $(4x+1)^2 \dfrac{d^2y}{dx^2} + 2(4x+1)\dfrac{dy}{dx} + y = 2x+1$

b) Solve $\dfrac{dx}{3z-4y} = \dfrac{dy}{4x-2z} = \dfrac{dz}{2y-3x}$. 5

3. a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y(x, 0) = y_o \sin^3\left(\dfrac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement y at any distance x from one end at any time t (use wave equation $\dfrac{\partial^2 y}{\partial t^2} = c^2 \dfrac{\partial^2 y}{\partial x^2}$). 8

b) System of differential equations of an undamped system is given by

$\ddot{y}_1 = -10y_1 + 4y_2$

$\ddot{y}_2 = 4y_1 - 4y_2$. 8

Assuming that there is no friction, find the natural frequencies of the system and corresponding normal modes of vibration using matrix method. Initial conditions are $y_1(0) = 0,\ y_2(0) = 0,\ \dot{y}_1(0) = \sqrt{2},\ \dot{y}_2(0) = 2\sqrt{2}$.

OR

4. a) Solve $\frac{\partial u}{\partial t} = K\frac{\partial^2 u}{\partial x^2}$ if

i) $u(0, t) = 0$

ii) $u_x(l, t) = 0$

iii) $u(x, t)$ is bounded and

iv) $u(x, 0) = \frac{u_0 x}{l}$ for $0 \le x \le l$. 8

b) A body weighing 20 Kg is hung from a spring. A pull of 40 Kg will stretch the spring to 10 cm. The body is pulled down to 20 cm below equilibrium position and then released. Find the displacement of the body from its equilibrium position in time t seconds, the maximum velocity and period of oscillation. 8

5. a) Find the Fourier sine and Fourier cosine transform of $f(n) = e^{-x}$. 6

b) Find the Laplace transform of the following **(any two)** : 6

i) $t^2 \sin 4t$

ii) $\frac{\cos at - \cos bt}{t}$

iii) $e^{-4t}\int_0^t t \sin 3t \, dt$. 6

c) Using Laplace transform solve following differential equation :

$$\frac{d^2x}{dt^2} + 9x = 18t, \; x(0) = 0, \; x\left(\pi/2\right) = 0.$$ 5

OR

6. a) Solve :

$$\int_0^\infty f(x)\cos\lambda x\,dx = e^{-\lambda}, \lambda > 0.$$ **6**

b) Find inverse Laplace transform of the following **(any two)** : **6**

i) $\cot^{-1}\left(\frac{s-2}{3}\right)$

ii) $\frac{s^2}{(s^2+a^2)^2}$

iii) $\frac{s^3}{s^4-a^4}$.

c) Find the Fourier transform of

$$f(x)=\begin{cases}1, & |x|<a \\ 0, & |x|>a.\end{cases}$$ **5**

SECTION – II

VII. A) The two regression lines are $2x-y+1=0$ and $3x-2y+7=0$. Find the mean of x and y. Also find the regression coefficients byx and bxy and the correlation coefficient. **6**

B) A candidate is selected for 3 posts in an interview. For the first post, there are 3 candidates, for the second post there are 4 candidates and for the third there are 2 candidates. What are the chances of his getting atleast one post ? **5**

C) A set of 5 coins is tossed 3200 times and the number of heads appearing each time is noted. The results are given below.

No. of heads	:	0	1	2	3	4	5
Frequency	:	80	570	1100	900	500	50

Test the hypothesis that the coins are unbiased.

[χ_5^2 at 5% level of significance is 11.07]. 6

OR

VIII. A) The Bombay Municipal Corporation installed 2000 bulbs in the streets of Bombay. If these bulbs have an average life of 1000 burning hours with a standard deviation of 200 hours, what number of bulbs might be expected to fail in the first 700 burning hours ?

Area for z = 1.5 is 0.067. 5

B) The first 4 moments of a distribution about the values 5 are 2, 20, 40 and 50. From the given information obtain the first 4 central moments, mean, standard deviation and coefficient of skewness and Kurtosis. 6

C) The following mistakes per page were observed in a book. Fit a Poisson distribution to the data.

No. of mistakes per page	:	0	1	2	3	4
No. of times mistakes occurred	:	211	90	19	5	0

6

IX. A) Find the directional derivative of $\phi = e^{2x} \cos yz$ at (0, 0, 0) in the direction of tangent to the curve x = a sint, y = a cost, z = at at $t = \frac{\pi}{4}$. 6

B) Show that the vector field $f(r)\bar{r}$ is always irrotational and determine f(r) such that the field is solenoidal also. 6

C) Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$. 4

OR

X. A) Prove the following **(any two)** : 8

i) $\bar{b} \times \nabla[\bar{a} \cdot \nabla \log r] = \frac{\bar{b} \times \bar{a}}{r^2} - \frac{2(\bar{a} \cdot \bar{r})}{r^4} (\bar{b} \times \bar{r})$.

ii) $\nabla \cdot \left[r \nabla \left(\frac{1}{r^n} \right) \right] = \frac{n(n-2)}{r^{n+1}}$.

iii) $\nabla^2 (r^n \log r) = [\, n(n+1) \log r + 2n + 1 \,]\, r^{n-2}$.

B) If $\bar{F}_1 = yz\bar{i} + zx\bar{j} + xy\bar{k}$ and $\bar{F}_2 = (\bar{a} \cdot \bar{r})\, \bar{a}$. Then show that $\bar{F}_1 \times \bar{F}_2$ is solenoidal. 4

C) Find the constants a and b so that the surface $ax^2 - byz = (a + 2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point (1, –1, 2). 4

XI. A) Use Divergence theorem to evaluate $\iint_s \bar{F} \cdot d\bar{s}$ where $\bar{F} = 4x\bar{i} - 2y^2\bar{j} + z^2\bar{k}$ and s is the surface bounding the region $x^2 + y^2 = 4$, z = 0 and z = 3. 6

B) Find the work done if a force $\bar{F} = 2x^2 y\bar{i} + 3xy\bar{j}$ displaces a particle in the XY plane from (0, 0) to (1, 4) along a curve $y = 4x^2$. 5

C) Using Green's theorem evaluate $\int_c (x^2 y dx + x^2 dy)$ where C is the boundary described counter clockwise of the triangle with vertices (0, 0), (1, 0) and (1, 1). 6

OR

XII. A) Use Stoke's theorem to evaluate $\int_C \overline{F} \cdot d\overline{r}$ where $\overline{F} = y^2\overline{i} + xy\overline{j} + xz\overline{k}$ and C is the bounding curve of the hemisphere $x^2 + y^2 + z^2 = 9$, $z > 0$ oriented in the positive direction. 6

B) Evaluate $\int_C \overline{F} \cdot d\overline{r}$ where $\overline{F} = (2xy + 3)\overline{i} + (x^2 - 4z)\overline{j} - 4y\overline{k}$ where C is any path joining (0, 0, 0) to (1, –1, 3). 5

C) Find the surfaces of equipressure in the case of steady motion of a liquid which has velocity potential $\phi = \log x + \log y + \log z$ and is under the action of force $\overline{F} = yz\overline{i} + zx\overline{j} + xy\overline{k}$. 6
