Total No. of Questions : 12] [Total No. of Printed Pages : 7

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F. E. Examination - 2010

ENGINEERING MATHEMATICS - I

(2003 Course)

Time : 3 Hours] [Max. Marks : 100

Instructions :

(1) From section I solve Q.1 or Q. 2, Q. 3 or Q. 4, Q. 5 or Q. 6. From section II solve Q. 7 or Q. 8, Q. 9 or Q. 10, Q. 11 or Q. 12.

(2) Answers to the ***two sections*** *should be written in* ***separate books****.*

(3) Neat diagrams must be drawn wherever necessary.

(4) Black figures to the right indicate full marks.

(5) Use of electronic pocket calculator is allowed.

(6) Assume suitable data, if necessary.

SECTION - I

Q.1) (A) Define normal form of a matrix. Reduce the following matrix to its normal form and hence find its rank, where **[06]**

$$A = \begin{bmatrix} 3 & -6 & 4 & -3 & 2 \\ 2 & -4 & 3 & 1 & 0 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{bmatrix}$$

(B) Given the linear transformation $Y = AX$, where

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 5 \\ 1 & 3 & 3 \end{bmatrix}, \text{ find the co-ordinates}$$

(x_1, x_2, x_3) corresponding to (2, 0, 5) in Y. **[05]**

(C) Verify Cayley - Hamilton Theorem for

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

and use it to find A^4. **[06]**

OR

Q.2) (A) Find the eigen values and the corresponding eigen vectors of the following matrix : **[07]**

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$$

(B) If A is an orthogonal matrix then show that A^{-1} and A^t are also orthogonal. **[04]**

(C) Examine for linear dependence, the system of vectors $X_1 = (1, 2, 3)$, $X_2 = (3, -2, 1)$, $X_3 = (1, -6, -5)$. If dependent, find relation between them. **[06]**

Q.3) (A) Show that the roots of $(x + 1)^6 + (x - 1)^6 = 0$ are given by $-i \cot \left(\frac{2r+1}{12}\right)\pi$, $r = 0, 1, 2, 3, 4, 5$. **[06]**

(B) Find real and imaginary parts of $\tanh^{-1}(x + iy)$. **[06]**

(C) Prove that $\log\left(\frac{a+ib}{a-ib}\right) = 2i\tan^{-1}\left(\frac{b}{a}\right)$ and hence evaluate

$\cos\left[i\log\left(\frac{a+ib}{a-ib}\right)\right]$. **[05]**

OR

Q.4) (A) A square lies above real axis in Argand's Diagram and two of its adjacent vertices are the origin and the point 5 + 6i. Find the complex numbers representing other vertices. **[06]**

(B) If $\tan(x + iy) = i$, where x and y are real, prove that x is indeterminate and y is infinite. **[06]**

(C) By considering principal value, express in the form a + ib the expression $\left(1+i\sqrt{3}\right)^{(1+i\sqrt{3})}$ **[05]**

Q.5) (A) Find the n^{th} derivative of the function $e^{2x}\sin x\cos x$. **[05]**

(B) If $x = \sin\theta$, $y = \sin 2\theta$, then prove that

$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2-4)y_n = 0$ **[06]**

(C) Prove that : $\frac{b-a}{1+b^2} < \left(\tan^{-1}b - \tan^{-1}a\right) < \frac{b-a}{1+a^2}$, if a < b. **[05]**

OR

Q.6) (A) If $y = \frac{\sinh^{-1}x}{\sqrt{1+x^2}}$, then prove that,

$(1+x^2)y_{n+2} + (2n+3)xy_{n+1} + (n+1)^2 y_n = 0$. **[06]**

(B) If $I_n = \frac{d^n}{dx^n}\left(x^n \log x\right)$, prove that

$I_n = n\,I_{n-1} + (n-1)!$ and hence show that

$I_n = n!\left[\log x + 1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots + \frac{1}{n}\right]$. **[06]**

(C) If $f(x) = \frac{1}{x^2}$, $g(x) = \frac{1}{x}$, prove that 'C' of Cauchy's Mean Value Theorem is the harmonic mean between a and b. **[04]**

SECTION - II

Q.7) (A) Determine the range of convergence of

$$\sum_{n=1}^{\infty} \frac{n+1}{2n+1} \cdot \frac{(x-3)^n}{2^n}.$$ **[05]**

(B) Discuss the convergence of **any one** of the following : **[04]**

(1) $\left(\frac{1}{4}\right)^2 + \left(\frac{1\cdot 5}{4\cdot 8}\right)^2 + \left(\frac{1\cdot 5\cdot 9}{4\cdot 8\cdot 12}\right)^2 + ...$

(2) $\frac{2\cdot 1^3 + 5}{4\cdot 1^5 + 1} + \frac{2\cdot 2^3 + 5}{4\cdot 2^5 + 1} + ... + \frac{2\cdot n^3 + 5}{4\cdot n^5 + 1} + ...$

(C) Attempt **any two** of the following : **[08]**

(1) Expand $\frac{x}{e^x - 1}$ up to x^4.

(2) Arrange in powers of x using Taylor's Theorem

$7 + (x + 2) + 3(x + 2)^3 + (x + 2)^4 - (x + 2)^5$

(3) Prove that $e^{\cos^{-1}x} = e^{\pi/2}\left[1 - x + \frac{x^2}{2} - \frac{x^3}{3} + ...\right]$

OR

Q.8) (A) Obtain the range of convergence of the series

$$\sum \frac{\sqrt{n}}{\sqrt{n^2+1}} x^n$$ **[05]**

(B) Discuss the convergence of **any one** of the following : **[04]**

(1) $\frac{1}{2} + \frac{2}{3}x + \left(\frac{3}{4}\right)^2 x^2 + \left(\frac{4}{5}\right)^3 x^3 + ...$

(2) $\frac{1}{(\log 2)^2} + \frac{1}{(\log 3)^2} + ... + \frac{1}{(\log n)^2} + ...$

(C) Attempt **any two** of the following : **[08]**

(1) Prove that

$$e^{e^x} = e\left[1 + x + x^2 + \frac{5}{6}x^3 + \frac{5}{8}x^4 + ...\right]$$

(2) Expand $(1 + x)^{(1+x)}$ upto term containing x^3.

(3) Using Taylor's Theorem show that :

$$\sqrt{1 + x + 2x^2} = 1 + \frac{x}{2} + \frac{7}{8}x^2 - \frac{7}{16}x^3 + ...$$

Q.9) (A) Attempt **any two** of the following : **[08]**

(1) Evaluate : $\lim_{x\to 0}\left[\frac{1}{x^2} - \cot^2 x\right]$

(2) Evaluate : $\lim_{x\to 0}\left(\frac{a^x + b^x + c^x}{3}\right)^{1/x}$

(3) Evaluate : $\lim_{x\to 0}\left[\frac{\sin^{-1}x - x}{x^3}\right]$

(B) If $x = e^r \cos\theta$, $y = e^r \sin\theta$ P.T. **[04]**

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = e^{-2r}\left[\left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{\partial u}{\partial \theta}\right)^2\right]$$

(C) If $u = \log(x^3 + y^3 - x^2y - xy^2)$ prove that : [05]

(1) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$

(2) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -3$

OR

Q.10) (A) Attempt **any two** of the following : [08]

(1) Find the values of a, b, c so that

$$\lim_{x\to 0} \frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2.$$

(2) Evaluate : $\lim_{x\to 1} \left(1 - x^2\right)^{1/\log(1-x)}$

(3) Evaluate : $\lim_{x\to 0} \log_{\tan x} \tan 2x$

(B) If $u = ax + by$, $v = bx - ay$, find the value of [04]

$$\left(\frac{\partial u}{\partial x}\right)_y \cdot \left(\frac{\partial x}{\partial v}\right)_v \cdot \left(\frac{\partial y}{\partial v}\right)_x \cdot \left(\frac{\partial v}{\partial y}\right)_u$$

(C) Find $\frac{dy}{dx}$ when $y^{x^y} = \sin x$. [05]

Q.11) (A) If u, v, w are the roots of equation

$(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ [06]

(B) Prove that error in calculating the power $W = V^2/R$ generated in the resistor is $\frac{V}{R^2}(2R\delta V - V\delta R)$. If there are errors of 1% and 2% in measuring the voltage V and resistance R, find % error in calculating of work $W = V^2/R$. [05]

(C) Examine for stationary values

$f(x, y) = \sin x + \sin y + \sin(x + y)$ **[05]**

OR

Q.12) (A) If $v^2 + w^2 = x$, $y = w^2 + u^2$, $z = u^2 + v^2$

prove that $JJ' = 1$. **[06]**

(B) Examine whether the functions

$u = x + y + z$

$v = x^2 + y^2 + z^2$

$w = x^3 + y^3 + z^3 - 3xyz$

are functionally dependent. If dependent find the relation between them. **[05]**

(C) Use Lagranges Method of Undertermined Multipliers to find the maximum and minimum distance of the point (3, 4, 12) from the sphere $x^2 + y^2 + z^2 = 1$. **[05]**
