Total No. of Questions—12] [Total No. of Printed Pages—8

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S.E. (Civil) (I Sem.) EXAMINATION, 2010

ENGINEERING MATHEMATICS—III

(2008 COURSE)

Time : Three Hours **Maximum Marks : 100**

N.B. :— (*i*) Answers to the two Sections should be written in separate answer-books.

(*ii*) In Section I, attempt Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**.
In Section II, attempt Q. No. **7** *or* Q. No. **8**, Q. No. **9** *or* Q. No. **10**, Q. No. **11** *or* Q. No. **12**.

(*iii*) Neat diagrams must be drawn whenever necessary.

(*iv*) Figures to the right indicate full marks.

(*v*) Use of non-programmable electronic pocket calculator is allowed.

(*vi*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Solve any *three* : [12]

(*i*) $\left(D^2 + 5D + 6\right) y = e^{e^x}$

(*ii*) $\left(D^3 - 25D\right) y = \sinh 5x$

(*iii*) $\left(D^4 - 1\right) y = \cos x$

(*iv*) $\left(D^2 + 3D + 2\right) y = \sin\left(e^x\right)$ (Use method of variation of parameters)

(*v*) $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = x \log x$.

(*b*) Solve : [5]

$$\frac{dx}{dt} + x - y = te^t$$

$$2y - \frac{dx}{dt} + \frac{dy}{dt} = e^t.$$

Or

2. (*a*) Solve any *three* : [12]

(*i*) $\left(D^2 + 2D + 1\right) y = \frac{e^{-x}}{x + 2}$

(*ii*) $\left(D^4 + 2D^2 + 1\right) y = x \cos x$

(*iii*) $\left(D^2 - D + 1\right) y = x^3 - 3x^2 + 1$

(*iv*) $\left(D^2 + 1\right) y = \sec x$ (Use method of variation of parameters)

(*v*) $(x + 2)^2 \frac{d^2y}{dx^2} + 3(x + 2) \frac{dy}{dx} + y = 4 \sin\left[\log(x + 2)\right]$.

(*b*) Solve : [5]

$$\frac{dx}{y^3x - 2x^4} = \frac{dy}{2y^4 - x^3y} = \frac{dz}{9z\left(x^3 - y^3\right)}.$$

3. (*a*) The differential equation satisfied by a beam, uniformly loaded with one end-fixed and second subjected to a tensile force P is given by :

$$\mathrm{EI}\frac{d^2y}{dx^2} - \mathrm{P}y = -\frac{\mathrm{W}}{2}x^2.$$

Show that the elastic curve for the beam under conditions $y = 0, \frac{dy}{dx} = 0$, when $x = 0$, is given by :

$$y = \frac{\mathrm{W}}{2\mathrm{P}}\left[x^2 + \frac{2}{n^2} - \frac{e^{nx}}{n^2} - \frac{e^{-nx}}{n^2}\right];$$

where $\mathrm{EI} = \frac{\mathrm{P}}{n^2}$. [8]

(*b*) A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y = a \sin\frac{\pi x}{l}$ from which it is released at time $t = 0$. Find the displacement $y(x, t)$ from one end. [9]

Or

4. (*a*) A 1 kg weight, suspended from a spring stretches it 4 cm. If the weight is pulled down 8 cm below the equilibrium position and then released,

(*i*) Set up a differential equation.

(*ii*) Find the position and velocity as function of time.

(*iii*) Find the amplitude, period and frequency of motion. [8]

(b) A homogeneous rod of conducting material of length 100 cm has its ends kept at zero temperature and the temperature initially is :

$$u(x, 0) = x, \quad 0 \le x \le 50$$
$$= 100 - x, \quad 50 \le x \le 100.$$

Find the temperature $u(x, t)$ at any time. [9]

5. (a) Solve the following system of equations by Gauss-Seidel method : [8]

$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$
$$x + y + 54z = 110.$$

(b) Using fourth order Runge-Kutta method, evaluate the value of y when $x = 1.1$ given that : [8]

$$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}, \ y(1) = 1.$$

Or

6. (a) Solve the following system by Cholesky's method : [8]

$$4x_1 + 2x_2 + 14x_3 = 14$$
$$2x_1 + 17x_2 - 5x_3 = -101$$
$$14x_1 - 5x_2 + 83x_3 = 155.$$

(b) Numerical solution of the differential equation $\frac{dy}{dx} = 2 + \sqrt{xy}$ is tabulated as :

x	y
1.0	1.0
1.2	1.6
1.4	2.2771
1.6	3.0342

Find y at $x = 1.8$ by Milne's Predictor-Corrector method taking $h = 0.2$. [8]

SECTION II

7. (a) The scores of two golfers A and B for 10 rounds each are given below. Find mean and standard deviation for both players A and B and hence find who is more consistent : [7]

A	B
58	84
59	56
60	92
54	65
65	86
66	78
52	44
75	54
69	78
52	68

(*b*) Calculate the correlation coefficient for the following weights (in kg) of husband x and wife (y) and comment on the result : [6]

x	y
65	55
66	58
67	72
67	55
68	66
69	71
70	70
72	50

(*c*) If a random variable has a Poisson distribution such that P(1) = P(2), find :

(*i*) Mean of the distribution and

(*ii*) P(4). [4]

Or

8. (*a*) The first four moments of a distribution about the values 5 are 2, 20, 40 and 50. From the given information obtain the first four central moments, mean, coefficient of skewness and kurtosis. [6]

(*b*) Two random variables have regression lines equations as $7x - 16y + 9 = 0$ and $5y - 4x - 3 = 0$. Calculate the coefficient of correlation, mean $\bar{x}$ and $\bar{y}$. [5]

(c) The probability that a pen manufactured by a company will be defective is $\frac{1}{10}$. If 12 such pens are manufactured, find the probability that :

(i) extractly two will be defective

(ii) none will be defective. [6]

9\. (a) A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 5$ where t is the time. Find the velocity and acceleration at time $t = 1$. [4]

(b) Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point P(1, 2, 3) in the direction of the line PQ, where Q is the point (5, 0, 4). [5]

(c) Evaluate (any *two*) :

(i) $\nabla . \left(r^3 \bar{r}\right)$

(ii) $\nabla^2 \left(r^2 + \log r\right)$

(iii) $\nabla . \left(\frac{\bar{a} \times \bar{r}}{r}\right)$. [8]

Or

10\. (a) Find the directional derivative of $xy^2 + yz^3$ at (2, –1, 1) along the line $2(x - 2) = y - 1 = z - 1$. [6]

(b) Show that the vector field given by :

$$\bar{F} = \left(3x^2y + y\sec^2 x\right] i + \left(x^3 + \tan x\right) j - 4zk$$

is conservative and find scalar field such that $\bar{F} = \nabla\phi$. [6]

(c) Prove that : [5]

$$\bar{b} \times \nabla \left[\bar{a} . \nabla \log r\right] = \frac{\bar{b} \times \bar{a}}{r^2} - \frac{2(\bar{a} . \bar{r})}{r^4} \left(\bar{b} \times \bar{r}\right).$$

11. (*a*) If

$$\bar{F} = (2x + y^2)\, i + (3y - 4x)\, j$$

then evaluate :

$$\int_C \bar{F} \cdot d\bar{r}$$

around the parabolic arc $y^2 = x$ joining (0, 0) and (1, 1). [5]

(*b*) Show that : [5]

$$\iiint_V 7r^4 dV = \iint_S r^4 \bar{r} \cdot \hat{n}\, dS.$$

(*c*) Apply Stokes' theorem to calculate :

$$\int_C 4y dx + 2z dy + by dz$$

where C is the curve of intersection of $x^2 + y^2 + z^2 = 6z$ and $z = x + 3$. [6]

Or

12. (*a*) Evaluate :

$$\iint_S 3(x^3 i + y^3 j + z^3 k) \cdot d\bar{S};$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. [6]

(*b*) Evaluate :

$$\iint_S (\nabla \times \bar{F}) \cdot d\bar{S};$$

where

$$\bar{F} = (x^3 - y^3)\, i - xyz\, j + y^3\, \hat{k}$$

and S is the surface $x^2 + 4y^2 + z^2 - 2x = 4$ above the plane $x = 0$. [5]

(*c*) Obtain the equation of the stream-lines in case of steady motion of fluid defined by : [5]

$$\bar{q} = (y - xz)\, i + (yz + x)\, j + (x^2 + y^2)\, \hat{k}.$$