Total No. of Questions—12] [Total No. of Printed Pages—8

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S.E. (E & T/C/Comp/IT/Elect/Instrum) EXAMINATION, 2010

ENGINEERING MATHEMATICS-III

(2003 COURSE)

Time : Three Hours **Maximum Marks : 100**

N.B. :— (*i*) From Section I, attempt Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**.

From Section II, attempt Q. No. **7** *or* Q. No. **8**, Q. No. **9** *or* Q. No. **10**, Q. No. **11** *or* Q. No. **12**.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Figures to the right indicate full marks.

(*v*) Use of electronic pocket calculator and steam tables is allowed.

(*vi*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Solve any *three* : [12]

(*i*) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = e^x \cosh 2x$

(*ii*) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^x \cos x$

(*iii*) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$

(iv) $(x+1)^2 \frac{d^2y}{dx^2} + (x+1)\frac{dy}{dx} = (2x+3)(2x+4)$

(v) $\frac{dx}{y^2} = \frac{dy}{x^2} = \frac{dz}{x^2y^2z^2}$

(b) An inductor of 0.5 henries is connected in series with a resistor of 6 ohms, a capacitor of 0.02 farads, a generator having alternative voltage 24 sin $10t$, $t > 0$. Find the charge in the circuit. [4]

Or

2. (a) Solve any *three* : [12]

(i) $\frac{d^3y}{dx^3} - 3\frac{dy}{dx} - 2y = x^3e^{-x}$

(ii) $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^2 + \log x$

(iii) $\frac{d^2y}{dx^2} + y = x \sin x$ (by variation of parameters)

(iv) $\frac{d^4y}{dx^4} + 6\frac{d^2y}{dx^2} + 8y = \cos 3x \cos x + 10$

(v) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 2^{-x}$

(b) Solve for x and y : [4]

$$\frac{dx}{dt} - wy = a \cos pt$$

$$\frac{dy}{dt} + wx = a \sin pt$$

3. (a) If $v = 4xy(x^2 - y^2)$, find its harmonic conjugate u. Also, find $f(z) = u + iv$ in terms of z. [6]

(b) If $f(z) = u(x, y) + iv(x, y)$ is analytic then show that $u(x, y)$ and $v(x, y)$ are harmonic functions of x and y. [5]

(c) Evaluate : [5]

$$\int_C \frac{4z^2 + z}{z(z^2 - 1)^2} \, dz, \quad C : |z - 1| = \frac{1}{2}.$$

Or

4. (a) Find the map of the circle $|z - i| = 1$ under the mapping $w = \frac{1}{z}$ into the w-plane. [5]

(b) Evaluate the following using residue theorem : [6]

$$\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z - 1)^2 (z - 2)} dz.$$

(c) Find the bilinear transformation, which maps the points 0, -1, i of the z-plane into the points 2, ∞, $\frac{1}{2}(5 + i)$ of the w-plane. [5]

5. (a) Find the Fourier cosine transform of : [6]

$$f(x) = \begin{cases} \cos x, & 0 < x < a \\ 0 \quad , & x > a \end{cases}$$

Write integral representation for $f(x)$.

(*b*) Solve the integral equation : [6]

$$\int_0^\infty f(x) \sin \lambda x \, dx = \begin{cases} \lambda^2, & 0 \le \lambda \le 1 \\ 0, & \text{otherwise} \end{cases}.$$

(*c*) Find the *z*-transform (any *two*) : [6]

(*i*) $ne^{-n} \sin an, \; n \geq 0$

(*ii*) $f(n) = \begin{cases} 2^n, & n \ge 0 \\ (1/3)^n, & n < 0 \end{cases}$

(*iii*) $f(n) = {}^mC_n, 0 \le n \le m.$

Or

6. (*a*) Find the inverse *z* transform (any *two*) : [8]

(*i*) $\dfrac{z^2}{(z-1)\left(z-\frac{1}{2}\right)^2}, |z| > 1$

(*ii*) $F(z) = \dfrac{z^2}{z^2+1}$ (by integral inversion method)

(*iii*) $\dfrac{3z^2+2z}{z^2+3z+2}, 1 < |z| < 2.$

(*b*) Solve the difference equation : [5]

$$f(n+1) + \frac{1}{4} f(n) = \left(\frac{1}{4}\right)^n, \; n \ge 0, f(0) = 0.$$

(*c*) For the function : [5]

$$f(x) = \begin{cases} \pi/2, & 0 < x < \pi \\ 0, & x > \pi \;\&\; x < 0 \end{cases}$$

Find the integral representation of $f(x)$.

SECTION II

7. (*a*) Find Laplace transforms of the following (any *two*) : [8]

(*i*) $e^{-3t} \int_0^t t \sin 2t \, dt$

(*ii*) $\dfrac{1 - \cos 3t}{t}$

(*iii*) $t \, U(t - 4) - t^3 \delta(t - 2)$.

(*b*) Evaluate : [4]

$$\int_0^\infty e^{-t} \frac{\sin t}{t} \, dt.$$

(*c*) Solve, using Laplace Transform method : [5]

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} = 9 \quad \text{with } y(0) = 0$$
$$y'(0) = 0.$$

Or

8. (*a*) Find Inverse Laplace Transforms of the following (any *two*) : [8]

(*i*) $\cot^{-1}\left(\dfrac{s - 2}{3}\right)$

(*ii*) $\dfrac{4s - 5}{s^2 - s - 2}$

(*iii*) $\dfrac{e^{-s}}{\sqrt{s + 1}}$

(*b*) Find inverse Laplace Transform of $\frac{s^2}{(s^2+a^2)^2}$ using convolution theorem. [4]

(*c*) Find the Laplace Transform of the periodical function $f(t)$ where : [5]

$$f(t) = \begin{cases} t \quad , & 0 < t < \pi \\ \pi - t, & \pi < t < 2\pi \end{cases} \quad f(t + 2\pi) = f(t).$$

9. (*a*) The position vector of a particle at time t is : [5]

$$\bar{r} = \cos(t-1)\,\hat{i} + \sinh(t-1)\,\hat{j} + mt^3\hat{k}$$

Find the value of m so that at time $t = 1$, the acceleration is normal to the position vector.

(*b*) Show that the vector field given by : [6]

$$\bar{F} = (y^2 \cos x + z^2)\,\hat{i} + (2y \sin x)\,\hat{j} + (2xz)\hat{k}$$

is irrotational. Find scalar field ϕ such that :

$$\bar{F} = \nabla\phi.$$

(*c*) Find the directional derivative of [5]

$$\phi = xy^2 + yz^3 \quad \text{at } (1, -1, 1)$$

along the direction normal to the surface $x^2 + y^2 + z^2 = 9$ at (1, 2, 2).

Or

10. (*a*) Establish the following : [6]

(*i*) $\nabla\left(\frac{\bar{a}\,.\,\bar{r}}{r^n}\right) = \frac{\bar{a}}{r^n} - \frac{n\,(\bar{a}\,.\,\bar{r})}{r^{n+2}}\,\bar{r}$

(*ii*) $\nabla\,.\left[r\,\nabla\left(\frac{1}{r^3}\right)\right] = \frac{3}{r^4}.$

(*b*) Find the function $f(r)$ so that $f(r)\ \bar{r}$ is solenoidal vector. [5]

(*c*) If directional derivative of : [5]

$$\phi = ax^2y + by^2z + cz^2x \text{ at } (1,\ 1,\ 1)$$

has maximum magnitude 15 in the direction parallel to

$$\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$$

find the values of a, b, c.

11. (*a*) Evaluate : [5]

$$\int_C \bar{F}\,.\,d\bar{r}$$

for $\bar{F} = (2x + y)\,\hat{i} + (3y - x)\hat{j}$

along the straight line joining (0, 0) and (3, 2).

(*b*) Evaluate $\iint_S (x^3\hat{i} + y^3\hat{j} + z^3\hat{k})\,.\,d\bar{S}$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 16$. [6]

(*c*) Verify Stoke's theorem for [6]

$$\bar{F} = xy^2\hat{i} + y\hat{j} + z^2x\hat{k}$$

for the surface of rectangular lamina bounded by $x = 0$, $y = 0$, $x = 1$, $y = 2$, $z = 0$.

Or

12. (*a*) Evaluate the surface integral : [6]

$$\iint_S (y^2z^2\, i + z^2x^2\hat{j} + x^2y^2\hat{k}) \,.\, d\bar{S}$$

where S is the upper part of sphere $x^2 + y^2 + z^2 = 4$ above the xoy plane.

(*b*) Use Stokes' theorem to evaluate : [6]

$$\iint_S (\nabla \times \bar{F}) \,.\, d\bar{S} \quad \text{where}$$

$$\bar{F} = (x^3 - y^3)\,\hat{i} - xy\,z\hat{j} + y^3\hat{k}$$

and S is the surface $x^2 + 4y^2 + z^2 - 2x = 4$ above the plane $x = 0$.

(*c*) Maxwell's equations are [5]

$$\nabla \,.\, \bar{B} = 0,\ \nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

Given $\bar{B} = \text{curl}\ \bar{A}$, then deduce that $\bar{E} + \dfrac{\partial \bar{A}}{\partial t} = -\ \text{grad}\ V$ where V is a scalar point function.