Total No. of Questions—12] [Total No. of Printed Pages—8+4

[3762]-24

S.E. (Mech./Prod. and S/W) EXAMINATION, 2010

ENGINEERING MATHEMATICS–III

(2003 COURSE)

Time : Three Hours **Maximum Marks : 100**

N.B. :— (*i*) Answer Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**. from Section I and Q. No. **7** *or* Q. No. **8**, Q. No. **9** *or* Q. No. **10**, Q. No. **11** *or* Q. No. **12** from Section II.

(*ii*) Answers to the *two* Sections should be written in separate answer-books.

(*iii*) Neat diagrams must be drawn whenever necessary.

(*iv*) Figures to the right indicate full marks.

(*v*) Use of electronic pocket calculator is allowed.

(*vi*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Solve any *three* of the following : [12]

(*i*) $(D^2 - 1)y = e^{-x} \sin(e^{-x}) + \cos(e^{-x})$

(*ii*) $(D^3 + D)y = \cos x$

P.T.O.

(*iii*) $(D^2 + 9)y = \frac{1}{1 + \sin 3x}$ (use method of variation of parameters)

(*iv*) $(1 + x)^2 \frac{d^2y}{dx^2} + (1 + x)\frac{dy}{dx} + y = 2 \sin \log (1 + x)$

(*v*) $(D^4 - 2D^3 - 3D^2 + 4D + 4)y = x^2e^x$.

(*b*) Solve :

$$\frac{dx}{dt} - 3x - 6y = t^2$$

$$\frac{dx}{dt} + \frac{dx}{dt} - 3y = e^t.$$ [4]

Or

2. (*a*) Solve any *three* of the following : [12]

(*i*) $(D^2 + 5D + 6)y = e^{ex}$

(*ii*) $\frac{d^2y}{dx^2} + 4y = x \sin x$

(*iii*) $(D^2 + 1)y = \sec x \tan x$ (Use method of variation of parameters) (*iv*) $(D^4 - 1)y = \cosh x \sinh x$

(*v*) $x^2 \frac{d^3y}{dx^3} + 3x\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{y}{x} = \log x$.

(*b*) Solve :

$$\frac{dx}{x(2y^4 - z^4)} = \frac{dy}{y(z^4 - 2x^4)} = \frac{dz}{z(z^4 - y^4)}.$$ [4]

3. (*a*) A string is stretched tightly between $x = 0$, $x = l$ and both ends are given displacement $y = a \sin pt$ perpendicular to the string. If the string satisfies the differential equation

$$\frac{\partial^2 y}{dx^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2},$$

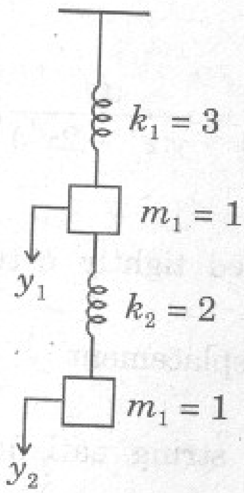
prove that the oscillations of the string are given by

$$y = a \sec\left(\frac{pl}{2c}\right)\cos\left(\frac{px}{c} - \frac{pl}{2c}\right)\sin pt.$$ [8]

(*b*) The system shown in the following figures begins to move with initial displacement :

$y_0 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and initial velocities

$\dot{y}_0 = \begin{bmatrix} -2\sqrt{6} \\ \sqrt{6} \end{bmatrix}$, assuming that there is no friction in the system, determine subsequent motion using eigen values. [8]



Or

4. (*a*) Solve the following equation

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2},$$

where $u(x, t)$ satisfies the following conditions :

(*i*) $u(0, t) = 0$

(*ii*) $u(l, t) = 0$ for all t

(*iii*) $u(x, 0) = x$ in $0 < x < l$

(*iv*) $u(x, t)$ is finite. [8]

(b) A body weighing 20 kg is hung from a spring. A pull of 40 kg will stretch the string to 10 cm, the body is pulled down 20 cm below equilibrium position and then released. Find the displacement of the body from its equilibrium position in time t second, the maximum velocity and period of oscillation. [8]

5. (a) Find Laplace transform of (any *two*) : [8]

(i) $t^2 \sin 4t$

(ii) $\dfrac{\cos at - \cos bt}{t}$

(iii) $e^{-4t} \int_0^t t \sin 3t \, dt$.

(b) Find the Fourier sine and cosine transform of $f(x) = e^{-x}$. [6]

(c) Using Laplace transform solve the differential equation :

$$\frac{d^2x}{dt^2} + 9x = 18t,$$

$$x(0) = 0,\ x\left(\frac{\pi}{2}\right) = 0.$$ [4]

Or

6. (*a*) Find the inverse Laplace transform of (any *two*) : [8]

(*i*) $\cot^{-1}\left(\frac{s-2}{3}\right)$

(*ii*) $\frac{s^2}{(s^2+a^2)^2}$

(*iii*) $\frac{s^3}{s^4-a^4}$.

(*b*) Solve :

$$\int_0^\infty f(x)\cos\lambda x\, dx = e^{-\lambda}, \lambda > 0.$$ [6]

(*c*) Find the Fourier transform of

$$f(x) = 1, \quad |x| < a$$
$$= 0, \quad |x| > a.$$ [4]

SECTION II

7. (*a*) The first four moments about the working mean 30.2 of a distribution are 0.255, 6.222, 30.211 and 400.25. Calculate the moments about the mean. Also calculate β_1, β_2. [5]

(b) Find the lines of regression for the following data : [7]

x	y
10	12
14	16
19	18
26	26
30	29
34	35
39	38

(c) A throw is made with two dice. Find the probability that :

(i) the sum is 7 or less

(ii) the sum is a perfect square. [4]

Or

8. (a) The mean weight of 500 students is 63 kg and the standard deviation is 8 kgs. Assuming that the weight are normally distributed. Find how many students weight 52 kg. The weights are recorded to the nearest kg.

Given, Area under 1.4375 = 0.4251

Area under 1.3125 = 0.4049. [5]

(b) A Manufacturer of cotter pins known that 2% of his produc is defective. If he sells cotter pins in boxes of 100 pins anc guarantees that not more than 5 pins will be defective ir a box. Find the approximate probability that a box will fai to meet the guranteed qualility. [5

(c) From a group of 10 students, marks obtained by each in papers of statistics and applied mechanics are given as :

x (Marks in Statistics)	**y** (Marks in App. Mechanics)
23	25
28	22
42	38
17	21
26	27
35	39
29	24
37	32
16	18
46	44

Calculate Karl Pearson's coefficient of correlation. [6]

(*b*) A Manufacturer of cotter pins known that 2% of his product is defective. If he sells cotter pins in boxes of 100 pins and guarantees that not more than 5 pins will be defective in a box. Find the approximate probability that a box will fail to meet the guranteed qualility. [5]

(*c*) From a group of 10 students, marks obtained by each in papers of statistics and applied mechanics are given as :

x **(Marks in Statistics)**	y **(Marks in App. Mechanics)**
23	25
28	22
42	38
17	21
26	27
35	39
29	24
37	32
16	18
46	44

Calculate Karl Pearson's coefficient of correlation. [6]

9. (a) Find the angle between the tangents to the curve $x = t^3 + 2$; $y = 4t - 5$, $z = 2t^2 - 6t$ at $t = 0$ and $t = 2$. [5]

(b) Show that the vector field given by

$$\bar{F} = (y^2 \cos x + xz^2)\hat{i} + 2y \sin x \,\hat{j} + 2xz\hat{k}$$

is irrotational, and find scalar potential field Q such that :

$$\bar{F} = \nabla Q .$$ [6]

(c) Show that the vector field $f(r)\,\bar{r}$ is always irrotational and determine $f(r)$ such that :

$$\nabla^2(f(r) = 0.$$ [6].

Or

10. (a) Find the tangential and normal component of acceleration of a particle moving on the curve $x = t^2 + 1$; $y = t^2$; $z = t$ at $t = 1$. [6]

(b) If the directional derivative of

$Q = axy + byz + czx$ at $(1, 1, 1)$

has maximum magnitude 5 in the direction parallel to y-axis, find the values of a, b, c. [5]

(c) Attempt any *two* of the following : [6]

(i) Prove that

$$\nabla(\varphi\nabla\psi - \psi\nabla\varphi) = \varphi\nabla^2\psi - \psi\nabla^2\varphi$$

(ii) Prove that :

$$\nabla(\bar{r} \, . \, \bar{u}) = \bar{r} \times (\nabla \cdot \bar{u}) + (\bar{r} \, . \, \nabla)\, \bar{u} + \bar{u} \, .$$

(iii) If $\bar{F}_1$ and $\bar{F}_2$ are irrotational then prove that $\bar{F}_1 \times \bar{F}_2$ is solenoidal.

11. (a) Find the work done by the force

$$\bar{F} = (2y + 3)\hat{i} + xz\hat{j} + (yz - x)\,\hat{k}$$

is moving a particle along the straight line joining (0, 0, 0) and (3, 1, 1). [6]

(b) Use Stokes theorem to evaluate :

$$\oint_E \bar{F} \, . \, d\bar{r}$$

where $\bar{F} = xy^2\hat{i} + 4\hat{j} + z^2x\hat{k}$ for the surface of rectangular lamina bounded by $x = 0$, $y = 0$, $z = 0$, $x = 1$, $y = 2$. [6]

(c) Find the surface of equipressure in case of steady motion of a liquid which has velocity potential

$$\phi = \log(xyz)$$

and is under the action of force $\overline{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$. [5]

Or

12. (a) Evaluate :

$$\int_C \overline{F} . d\overline{r}$$

for $\overline{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ along the curve $x = 2t^2 : y = t;$ $z = 4t^2 - t$ from $t = 0$ to $t = 1$. [5]

(b) Evaluate :

$$\iint \overline{F} . \hat{n}\, ds$$

over the region of surface of the tetrahedron bounded by co-ordinate planes and the plane

$$2x + y + 2z = 6$$

where, $\overline{F} = (x + y^2)\hat{i} - 2x\hat{j} + 2yz\hat{k}$. [6]

(*c*) Apply Stokes theorem to calculate

$$\int_{C} 4y\,dx + 2z\,dy + 6y\,dz,$$

where, C is the curve of intersection of $x^2 + y^2 + z^2 = 6z$ and $z = x + 3$. [6]