6

Total No of Questions: [12]	*SEAT NO. :*	

[Total No. of Pages : 6]

B.E. Mechanical (2008 Pattern)

Finite Element Method

(Elective - III) (Semester - II)

Time: 3 Hours *Max. Marks : 100*

Instructions to the candidates:

1) *Answers to the two sections should be written in separate answer books.*
2) *Neat diagrams must be drawn wherever necessary.*
3) *Figures to the right side indicate full marks.*
4) *Use of Calculator is allowed.*
5) *Assume Suitable data if necessary.*
6) *Additional data sheet is attached for the reference.*

SECTION I

Unit I

Q1)	a)	Explain how banded skyline solution method is used to solve simultaneous equations.	[8]
	b)	List at least 6 advantages of Finite Element Method over analytical method. Also list disadvantages or limitations of FEM.	[8]

OR

Q2)	a)	Explain the terms essential and natural boundary conditions. Give example of each.	[8]
	b)	Explain in detail the method of matrix partitioning and how it is used to impose boundary conditions in finite element method.	[8]

Unit II

Q3) a) For the plane truss shown in Figure 3a, determine the following. Each element has E = 70 GPa, and area A = 200 mm^2. [10]

i. write down the elemental stiffness matrices (k) for each element,
ii. assemble k matrices to get global stiffness matrix (K),
iii. apply boundary conditions,
iv. find horizontal and vertical displacements of node 1,
v. determine reaction forces at node 2, 3 and 4.

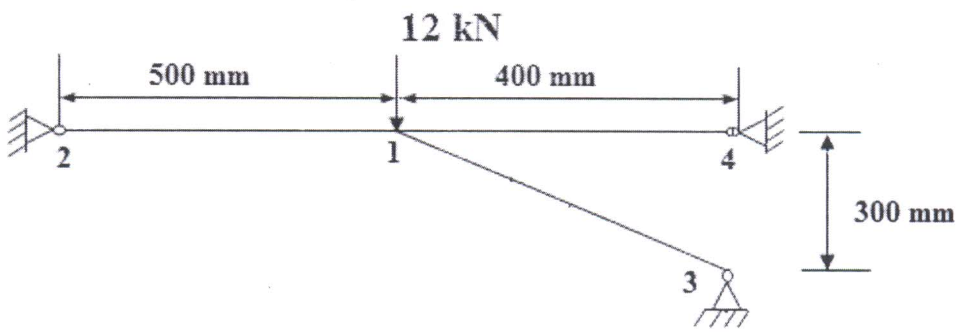
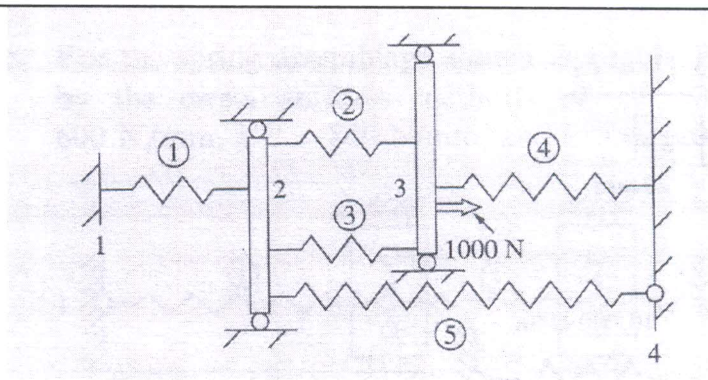
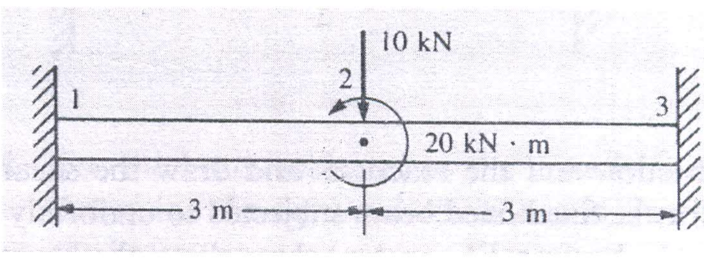


Figure 3a

b) For the five spring assemblage shown in Figure 3b, determine the displacements at nodes 2 and 3 and the reactions at nodes 1 and 4. Assume the rigid vertical bars at nodes 2 and 3 connecting the springs remain horizontal at all times but are free to slide or displace left or right. There is an applied force at node 3 of 1000 N to the right. Consider k1 = 500N/mm, k2 = k3 = 300N/mm, and k4 = k5 = 400N/mm [8]

		 Figure 3b	
		OR	
Q4)	a)	Derive elemental stiffness matrix and force vector for two-nodded (linear) bar element using Principle of Minimum Potential Energy (PMPE) Method. The bar element is oriented in x-direction. Assume that only concentrated forces are acting on the nodes of bar element. Ignore surface tractions and the body forces.	[10]
	b)	For the beam shown in Figure 4b, determine the displacements and the slope at node 2. Also find the reaction forces and moments at nodes 1 and 3. Consider E = 210 GPa, and I = 4 X 10^{-4} m^4  Figure 4b	[8]
		Unit III	
Q5)		Evaluate the following integrals.	
	a)	use two-point Gaussian quadrature method $$I = \int_0^1 \frac{1}{1+x^2} dx$$	[5]
	b)	use two-point Gaussian quadrature method $$I = \int_1^5 [3^x + 1] dx$$	[5]
	c)	use three-point Gaussian quadrature method $$I = \int_4^5 \frac{2\sin(x)}{x^2} dx$$	[6]
		OR	
Q6)	a)	Explain with neat sketches the difference between p and h refinements in Finite Element Method.	[6]

b) For the three-nodded iso-parametric bar element shown in Figure 6b, show that the Jacobean determinate is $|J| = L/2$. Also determine the shape functions $N1$ - $N3$ and the strain-displacement matrix $[B]$. Assume the displacement field as $u = a1 + a2s + a3s^2$. [10]

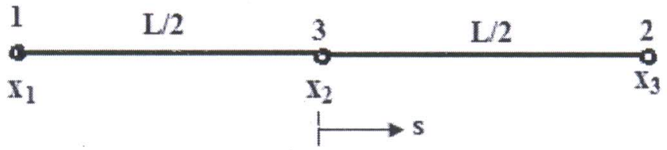


Figure 6b

SECTION II

Unit IV

Q7) Determine the temperature distribution along the length of the rod (at L/4, L/2, 3L/4, and L) as shown in Figure 7. The rod with radius of 25 mm is insulated at the perimeter. The left end has a constant temperature of 40 °C and a free stream temperature T_∞ is -10 °C. Let $Kxx = 35$ W/(m °C) and $h = 55$ W/(m^2 °C). [16]

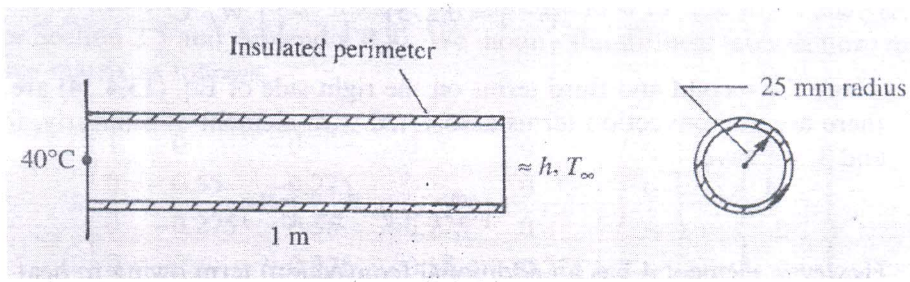


Figure 7

OR

Q8) A composite wall shown in Figure 8 is composed of two homogeneous slabs in contact. Let thermal conductivities be $K1 = 1$ W/(m °C) for firebrick slab 1 and $K2 = 0.3$ W/(m °C) for insulating slab 2. The left side is exposed to an ambient temperature of $T_{\infty L} = 1000$ °C inside the furnace with heat transfer coefficient of $hL = 10$ W/(m^2 °C). The right side ambient temperature is $T_\infty R = 25$ °C outside of the furnace with heat transfer coefficient of $hR = 3$ W/(m^2 °C). The thicknesses of the slabs are $L1 = 0.20$ m, and $L2 = 0.10$ m. Determine the temperature at the left edge, point between the two slabs and right edge of the composite wall. [16]

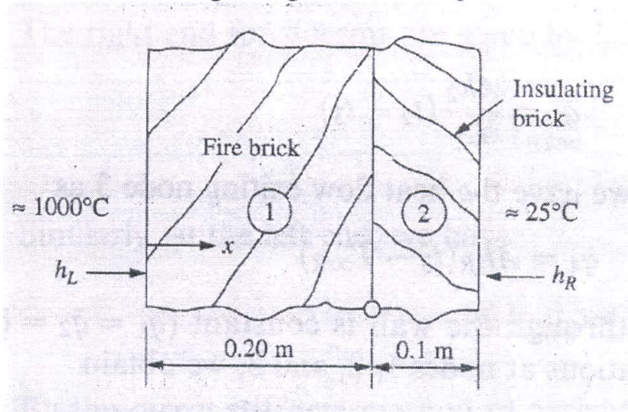


Figure 8

Unit V

9) a) For the stepped bar shown in Figure 9a, determine the first two natural frequencies in terms of rad/s for un-damped free vibration. Let $A_1 = A_3 = 5\ cm^2$, $A_2 = 10\ cm^2$, $E = 210$ GPa, and $\rho = 7860\ kg/m^3$. Use consistent mass matrices for each element. [12]

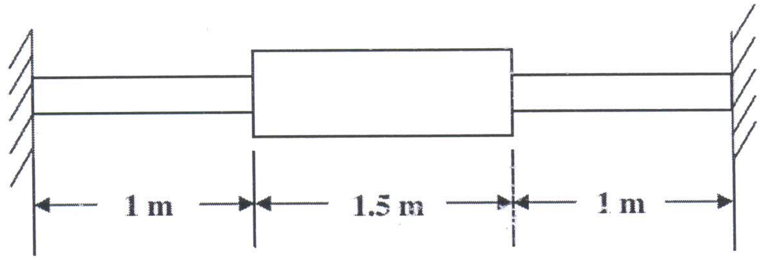


Figure 9a

b) Explain eigenvalue problem for un-damped free vibration system [6]

OR

Q10) a) For the tapered beam with square cross sectional area shown in Figure 10, set-up the problem to find characteristic equation to determine first four natural frequencies. Consider two elements of average cross sectional area and equal lengths. Each element has E = 200 GPa, density $\rho = 7500$ kg/m3, $A_1 = 9\ cm^2$, and
$A_2 = 4\ cm^2$. Use consistent mass matrix. [18]

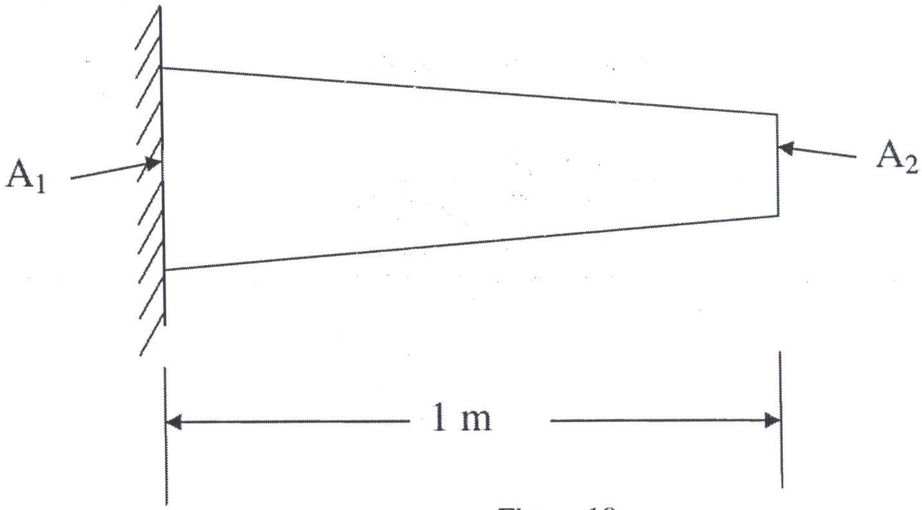


Figure 10

Unit VI

Q11) a) What are various meshing techniques? [8]

b) Explain the terms strain and stress recovery in post-processing stage. [8]

OR

Q12) Consider the truss problem shown in Figure 12 for the calculation of displacements. For this problem write down nodal coordinates, element connectivity, type of analysis, loading and boundary conditions. [16]

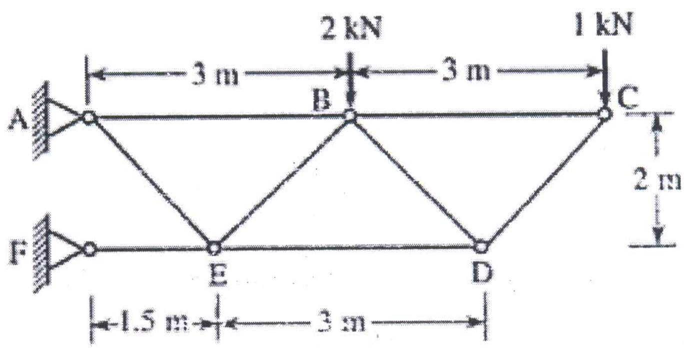


Figure 12

DATA SHEET

Shape Functions:

1 Bar Element:

$$N_1 = 1 - \frac{x}{L} \qquad N_2 = \frac{x}{L}$$

2 Beam Element:

$$N_1 = \frac{1}{L^3}(2x^3 - 3x^2L + L^3)$$
$$N_2 = \frac{1}{L^3}(x^3L - 2x^2L^2 + xL^3)$$
$$N_3 = \frac{1}{L^3}(-2x^3 + 3x^2L)$$
$$N_4 = \frac{1}{L^3}(x^3L - x^2L^2)$$

Elemental Stiffness Matrices:

1 Bar Element:

$$k_{bar} = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

2 Beam Element:

$$k_{beam} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

3 Truss Element:

$C = \cos(\theta)$ and $S = \sin(\theta)$

θ is positive in anti clockwise direction.

$$k_{truss} = \frac{AE}{L}\begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix}$$

Elemental Mass Matrices:

1 Bar Element:

(a) Consistent mass matrix:

$$m_{consistent} = \frac{\rho AL}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

(b)	Lumped mass matrix:	
	$$m_{lumped} = \frac{\rho AL}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$	
2	Beam Element:	
(a)	Consistent mass matrix:	
	$$m_{consistent} = \frac{\rho AL}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix}$$	
(b)	Lumped mass matrix:	
	$$m_{lumped} = \frac{\rho AL}{2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$	

Heat Transfer Matrices:

k matrix for Conduction + Convection for bar element:

$$k = \frac{AK}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{hPL}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

where, A = cross sectional area, K = Thermal Conductivity, L = Length of an element, h = Convection Coefficient, and P = Perimeter.

Gauss Quadrature:

Table for Gauss Points for integration from -1 to 1

$$\int_{-1}^{1} y(x)dx = \sum_{i=1}^{n} W_i y_i$$

Number of Points	Locations, x_i	Associated Weights, W_i
1	$x_1 = 0.000$	2.000
2	$x_1, x_2 = \pm\ 0.57735$	1.000
3	$x_1, x_3 = \pm\ 0.77459$ $x_2 = 0.000$	$5/9 = 0.55556$ $8/9 = 0.88889$
4	$x_1, x_4 = \pm\ 0.86113$ $x_2, x_3 = \pm\ 0.33998$	0.34785 0.65214