Total No. of Questions : 12]

SEAT No. :

P2022

[Total No. of Pages : 5

F.E.

ENGINEERING MATHEMATICS - I

(2008 Pattern)

Time : 3 Hours] *[Max. Marks :100*

Instructions to the candidates:

1) *Answer Q.1 or Q.2, Q.3 or Q.4, Q.5 or Q.6 from section - I and Q.7 or Q.8, Q.9 or Q.10, Q.11 or Q.12 from section - II.*
2) *Answers to the two sections should be written in separate answer books.*
3) *Figures to the right indicate full marks.*
4) *Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.*
5) *Assume suitable data, if necessary.*

SECTION - I

Q1) a) Reduce the following matrix to normal form and hence find its rank.

$$A=\begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}$$ **[5]**

b) Find for what value of k, the set of equations **[6]**

$$2x - 3y + 6z - 5t = 3$$
$$y - 4z + t = 1$$
$$4x - 5y + 8z - 9t = k$$

has(i) No solution (ii) An infinite number of solutions.

c) Verify Caley - Hamilton theorem for the following matrix and use it to find A^{-1}. **[7]**

$$A=\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$$

OR

Q2) a) Find Eigen values and the corresponding Eigenvectors of the following matrix. **[6]**

$$A=\begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

b) Examine whether the following vectors are linearly dependent. If so find the relation between them: **[6]**

$\overline{x}_1 = (3, 1, -4)$ $\quad \overline{x}_2 = (2, 2, -3)$ $\quad \overline{x}_3 = (0, -4, 1)$

c) For the given transformation **[6]**

$$Y=\begin{bmatrix} 4 & -5 & 1 \\ 3 & 1 & -2 \\ 1 & 4 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

find the coordinates (x_1, x_2, x_3) corresponding to (2, 9, 5) in Y.

Q3) a) Find all values of $(1 + i)^{1/5}$. Show that their product is $1 + i$. **[5]**

b) Find z if $\arg(z + 2i) = \pi/4$, $\arg(z - 2i) = 3\pi/4$. **[5]**

c) If $\tan(\alpha + i\beta) = x + iy$ prove that **[6]**

$$x^2 + y^2 + 2x \cot 2\alpha = 1 \text{ and } x^2 + y^2 - 2y \coth\beta = -1$$

OR

Q4) a) Solve the equation $x^9 - x^5 + x^4 - 1 = 0$ using De Moivre's theorem. **[5]**

b) A square lies above real axis in Argand diagram, and two of its adjacent vertices are the origin & the point 5 + 6i. Find the complex numbers representing other two vertices of the square. **[6]**

c) Prove that $\tan\left\{i\log\left(\frac{a-ib}{a+ib}\right)\right\}=\frac{2ab}{a^2+b^2}$. **[5]**

Q5) a) Find n^{th} derivative of $y = e^x \cos x \cos 2x$. **[5]**

b) If $y = e^{m\cos^{-1}x}$ prove that **[5]**

$$(1 - x^2)\, y_{n+2} - (2n + 1)x\, y_{n+1} - (n^2 + m^2)y_n = 0$$

c) Discuss convergence of the following series (any one) : **[6]**

i) $\frac{1}{1+z}+\frac{2}{1+z^2}+\frac{3}{1+z^3}+\ldots\ldots\ldots$

ii) $\sum_{n=1}^{\infty}\left(\frac{n+1}{n^3}\right)x^n \quad , x > 0$

OR

Q6) a) Find n^{th} derivative of $y = \dfrac{x}{(x-1)(x-2)(x-3)}$. **[5]**

b) If $y = \left(x + \sqrt{x^2 - 1}\right)^m$ show that

$(x^2 - 1)y_{n+2} + (2n + 1)x\, y_{n+1} + (n^2 - m^2)y_n = 0$ **[5]**

c) Discuss convergence of the following series (any one) : **[6]**

i) $1 + \dfrac{3}{7}x + \dfrac{3.6}{7.10}x^2 + \dfrac{3.6.9}{7.10.13}x^3 + \ldots\ldots\ldots\ldots$

ii) $\sum_{n=1}^{\infty} \dfrac{n^2}{n!}$

SECTION - II

Q7) a) Expand $\log \cos\left(x + \dfrac{\pi}{4}\right)$ using Taylor's theorem in ascending power of x up to x^3. **[5]**

b) Obtain the Maclaurin's expansion of $\tan\left(\dfrac{\pi}{4} + x\right)$ and hence find the value of tan(46.5°) to three places of decimals. **[5]**

c) Solve (any one)

i) Find a and b if

$$\lim_{x \to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3} = 1$$

ii) Prove that **[6]**

$$\lim_{x \to \infty} \left(\frac{ax + 1}{ax - 1}\right)^x = e^{2/a}$$

OR

Q8) a) Using Taylor's theorem express **[5]**

$7 + (x + 2) + 3(x + 2)^3 + (x + 2)^4$ in ascending powers of x.

b) Obtain the expansion of $\sinh^{-1}(3x + 4x^3)$ upto a term in x^5. **[5]**

c) Solve (any one) **[6]**

i) $\lim_{x \to y} \dfrac{x^y - y^x}{x^x - y^y}$.

ii) $\lim_{x \to \infty} \left\{ x - x^2 \log\left(1 + \dfrac{1}{x}\right) \right\}$.

Q9) Solve : (Any two) **[16]**

a) If $v = (1 - 2xy + y^2)^{-1/2}$ prove that

i) $x\dfrac{\partial v}{\partial x} - y\dfrac{\partial v}{\partial y} = y^2 v^3$

ii) $\dfrac{\partial}{\partial x}\left[(1 - x^2)\dfrac{\partial v}{\partial x}\right] + \dfrac{\partial}{\partial y}\left[y^2 \dfrac{\partial v}{\partial y}\right] = 0$.

b) If $x = u \tan v$, $y = u \sec v$

prove that

$$\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial v}{\partial x}\right)_y = \left(\frac{\partial u}{\partial y}\right)_x \left(\frac{\partial v}{\partial y}\right)_x$$

c) If $u = \sin^{-1}\sqrt{x^2 + y^2}$ prove that

i) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \tan u$

ii) $x^2 \dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2 \dfrac{\partial^2 u}{\partial y^2} = \tan^3 u$

OR

Q10) Solve : (Any two) **[16]**

a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ show that

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$$

b) If $u = \operatorname{cosec}^{-1}\sqrt{\dfrac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}}$ then, show that

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x \partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{\tan u}{12}\left[\frac{13}{12}+\frac{\tan^2 u}{12}\right]$$

c) If $u = x \log xy$ where

$x^3 + y^3 + 3xy = 1$ find $\dfrac{du}{dx}$.

Q11) a) Show that JJ' = 1 for **[6]**

$x = e^v \sec u$ and $y = e^v \tan u$

b) In calculating the volume of a right circular cone, errors of 2% and 1% are made in measuring the height and radius of base respectively. Find the error in the calculated volume. **[6]**

c) Determine the points where the function $x^3 + y^3 - 3axy$ has a maximum or minimum. **[6]**

OR

Q12) a) If $x = a(u + v)$, $y = b(u - v)$ and $u = r^2\cos 2\theta$, $v = r^2 \sin 2\theta$ using chain rule find $\dfrac{\partial(x, y)}{\partial(r,\theta)}$. **[6]**

b) Examine whether u,v are functionally dependent. If so find the relation between them. **[6]**

$$u = \frac{x+y}{1-xy},\ v = \tan^{-1}x + \tan^{-1}y$$

c) If $u = xyz$, $v = x^2 + y^2 + z^2$ and $w = x + y + z$ then find $\dfrac{\partial x}{\partial u}$. **[6]**

☆☆☆