Total No. of Questions : 12]

SEAT No. : ☐

P2020

[Total No. of Pages : 6

F.E. (Semester - II)
ENGINEERING MATHEMATICS - II
(2008 Course)

Time : 3 Hours] *[Max. Marks : 100*

Instructions to the candidates:

1) *In section - I Solve Q.1 or Q.2, Q.3 or Q.4, Q.5 or Q.6. In section-II Solve Q.7 or Q.8, Q.9 or Q.10, Q.11 or Q.12.*
2) *Neat diagrams must be drawn wherever necessary.*
3) *Figures to the right indicate full marks.*
4) *Use of non - programmable electronic pocket calculator is allowed.*
5) *Assume suitable data, if necessary.*

SECTION - I

Q1) a) Form the differential equation whose general solution is $y = ae^{2x} + be^{3x}$. **[6]**

b) Solve any two **[10]**

i) $\dfrac{dy}{dx} = \dfrac{y+2}{x+y+1}$

ii) $(3xy + 8y^5)dx + (2x^2 + 24xy^4)dy = 0$

iii) $\dfrac{dy}{dx} = \dfrac{y}{2y\log y + y - x}$

OR

Q2) a) Form the differential equation whose general solution is $y = (C_1 + C_2 t)e^{t}$ **[6]**

b) Solve any two **[10]**

i) $\dfrac{dy}{dx} = 1 - x\tan(x - y)$

ii) $(xy - 2y^2)dx - (x^2 - 3xy)dy = 0$

iii) $(x + \tan y)dy = \sin 2y \ dx$

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Q3) Solve any three: **[18]**

a) Radium decomposes at the rate proportional to the amount present. If 5% of the original amount disappears in 50 years, how much will remain after 100 years.

b) A constant e.m.f E volts is applied to a circuit containing a constant resistance R ohms in series and a constant inductance L henries. If the initial current is zero, show that the current builds upto half its theoretical maximum in $\left(\frac{L \log 2}{R}\right)$ seconds

c) A body at temperature 100° C is placed in a room whose temperature is 20° C and cools to 60° C in 5 minutes. Find its temperature after 10 minutes.

d) Find the orthogonal trajectories of $r = a(1-\cos\theta)$

OR

Q4) Solve any three: **[18]**

a) The inner and outer surfaces of a spherical shell are maintained at T_0 and T_1 temperatures respectively. If the inner and outer radii are r_0 and r_1 respectively and thermal conductivity of the shell is k, Find the amount of heat lost from the shell per unit time. Find also the temperature distribution through the shell.

b) A body starts moving from rest and is opposed by a force per unit mass of value cx and resistance per unit mass of value bv^2, where x is displacement and v is velocity of the body at that instant. show that the velocity of the particle is given by $v^2 = \frac{c}{2b^2}(1-e^{-2bx}) - \frac{cx}{b}$ e^{-2bx}

c) If the population of a country doubles in 50 years in how many years will it become three times under the assumption that the rate of increase is proportional to the number of inhabitants ?

d) A body originally at 80° C cools down to 60° C in 20 minutes, the temperature of air being 40° C. What will be temperature of the body after 80 minutes from the original?

Q5) a) Find the Fourier series to represent $f(x)=x^2$ in the interval $-l<x<l.$ **[9]**

Hence deduce:

$$\frac{\pi^2}{12}=\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+-----$$

b) Find the reduction formula for, **[7]**

$$I_n = \int_0^\infty e^{-x}\sin^n x\,dx$$

Hence show : $I_4 = \frac{24}{85}$

OR

Q6) a) Express 'y' as a Fourier series upto first harmonic where y is given as: **[8]**

x	0	$\frac{\pi}{3}$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$
y	1.0	1.4	1.9	1.7	1.5	1.2

b) Evaluate : $\int_0^\infty \frac{x^5}{5^x}dx$ **[4]**

c) Prove that : **[4]**

$$B(m,n) = B(m,n+1) + B(m+1,n)$$

SECTION - II

Q7) a) Trace the following curves (any two) **[8]**

i) $x\left(x^2+y^2\right)=a\left(x^2-y^2\right)a>0$

ii) $r=a\sin 2\theta$

iii) $x=a\left(t+\sin t\right), y=a\left(1-\cos t\right)$

b) Find the length of one loop of lemniscate **[5]**

$r^2=a^2\cos 2\theta$

c) $if\,\phi(a)=\int_a^{a^2}\frac{\sin ax}{x}dx$ find $\frac{d\phi}{da}$ **[4]**

OR

Q8) a) Trace the following curves (any two) **[8]**

i) $a^2y^2=x^2\left(a^2-x^2\right)$

ii) $r=\frac{a}{2}(1-\sin\theta)$

iii) $x^{2/3}+y^{2/3}=a^{2/3}$

b) Show that $\frac{d}{dt}\text{erf}\sqrt{t}=\frac{e^{-t}}{\sqrt{\pi t}}$ **[4]**

c) In Astroid $x^{2/3}+y^{2/3}=a^{2/3}$, arc length S is measured from cusp, which lies on y - axis, Show that $S^3\ \alpha\ x^2$ **[5]**

Q9) a) Find the equation of circle which is the section of sphere $x^2 + y^2 + z^2 + 6y - 6z - 21 = 0$ and has center on $M(2,-1,2)$ **[6]**

b) Find the equation of right circular cone with vertex $(1,2,-3)$ and semivertical angle $\cos^{-1}\frac{1}{\sqrt{3}}$, whose axis is $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z+1}{-1}$ **[5]**

c) Find the equation of right circular cylinder with radius 5 and axis is the line $\frac{x-2}{2} = \frac{y-3}{1} = \frac{z+1}{1}$ **[6]**

OR

Q10) a) A sphere with constant radius r passes through origin and meets co - ordinate axes in A, B, C respectively, Show that the locus of centroid of the triangle ABC is a Sphere $9\,(x^2 + y^2 + z^2) = 4\,r^2$ **[6]**

b) Find the equation of right circular cone passing through (1,1,2) with vertex as origin and axis is $6x = -3y = 4z$. **[5]**

c) Find the equation of right circular cylinder with axis **[6]**

$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$ and radius = 2

Q11) Solve any two **[16]**

$e^{-x^2-y^2}$

a) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} e^{-x^2-y^2}\,dx\,dy$

b) Prove that the mean distance of the points within a circular area of radius a, from a fixed point on the circumference is $\frac{32a}{9\pi}$

c) Find the volume of the region bounded by $z = x^2 + y^2$ and $z = 2x$

OR

Q12) Solve any two **[16]**

a) Evaluate $\iiint\limits_{V} \frac{dxdydz}{(1+x+y+z)^3}$

Over the volume of tetrahedron bounded by

$x=0, y=0, z=0$ and $x+y+z=1$

b) Find the centre of Gravity of the area in first quadrant, bounded by

$\left(\frac{x}{a}\right)^{2/3}+\left(\frac{y}{b}\right)^{2/3}=1$

Where density $\rho = \lambda xy, \lambda =$ constant

c) Find the area between the parabola $\frac{y+8}{x}=x-2$ and X $-$axis.