Total No of Questions: [12] **SEAT NO. :**

[Total No. of Pages : 04

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S.E. Civil (2008 Course)

Engg Mathematics - III

Time: 3 Hours **Max. Marks : 100**

Instructions to the candidates:

1) *Answers to the two sections should be written in separate answer books.*
2) *Answer Q1 or Q2, Q3 or Q4, Q5 or Q6 from section I, Q7 or Q8, Q9 or Q10, Q11 or Q12 from section II*
3) *Neat diagrams must be drawn wherever necessary.*
4) *Figures to the right side indicate full marks.*
5) *Use of Calculator is allowed.*
6) *Assume Suitable data if necessary*

SECTION I

Q1) a) Solve any three [12]

i) $(D^2 - 4D + 3)y = x^3 e^{2x}$

ii) $(D^3 - 6D^2 + 12D + 8)y = (x^2 + e^{2x} + \cos 2x)$

iii) $(D^2 - 3D + 2)y = \frac{1}{e^x} + \cos(\frac{1}{e^x})$

iv) $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$ (By Variational approach)

b) $\frac{du}{dx} + v = \sin x,\ \frac{dv}{dx} + u = \cos x,$ [05]

Q2) a) Solve any three: [12]

i) $(D^2 + 2D + 1)y = xe^x \cos x$

ii) $(D^2 - 1)y = \frac{2}{1+e^x}$

iii) $(D^2 + 6D + 9)y = \frac{1}{x^3} e^{-3x}$

iv) $(3x+2)^2 \frac{d^2y}{dx^2} + (3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$ So

b) Solve : $\frac{dx}{2x} = \frac{dy}{-y} = \frac{dz}{4xy^2 - 2z}$ [X]

Q3) a) A tightly stretched string with fixed points x=0 and x= l initially. In a position given [08]

by $y(x,0)=y_0 \sin^3(\frac{\pi x}{l})$, if it is released from rest, from this position, find the displacement y at any instant x from one end and at any time t.

b) The differential equation satisfied by a beam uniformly loaded with one end fixed and second subjected to a tensile force P is given by: [08]

$E|\frac{d^2y}{dx^2} - Py = -\frac{W}{2}x^2$. Show that the elastic curve for the beam under the conditions y=0, $\frac{dy}{dx}=0$ when x=0 is given by

$$y=\frac{W}{2P}\left[x^2+\frac{2}{n^2}-\frac{e^{nx}}{n^2}-\frac{e^{-nx}}{n^2}\right]$$ where El = P/n^2

Q4) a) Solve $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ if, [08]

i) u is finite, ii) u(0,t)-=0 iii) u(π,t)=0

iv) $u(x,0) = \pi x - x^2 \quad 0<x<\pi$

b) The differential equation for the displacement y of a heavy whiriling shaft is [08]

$\frac{d^4y}{dx^4}=a^4\left(y+\frac{g}{w^2}\right)$ where $a^4=\frac{Ww^2}{gEI}$. If both ends are in short bearings, the ends being x=0 and x=l, find the bending moment of the centre of the shaft.

Q5) a) Solve the following system of equations by gauss Seidel method [09]

$$28x+4y-z=32$$

$$x+3y+10z=24$$

$$2x+17y+4z=35$$

b) Numerical Solution of the differential equation $2\frac{dy}{dx}=(1+x^2)y^2$ is tabulated as [08]

X	0	0.1	0.2	0.3
Y	1	1	1.06	1.21

Evaluate y at x=0.4 and 0.5 by Milne's predictor –corrector method

OR

Q6) a) Solve the following system of equations by Cholesky's method [09]

$$4x_1 + 2x_2 = 0$$

$$-2x_1 + 4x_2 - x_3 = 1$$

$$-x_2 + 4x_3 = 0$$

b) Use Ranga Kutta method of fourth ordxer to find y(0,1) , given that, [08]

$$\frac{dy}{dx} = \frac{1}{x+y}, \quad y(0)=1$$

SECTION _ II

Q7) a) Calculate the first four moments about the mean of the distribution. Also find β_1 and β_2 [06]

X	2.0	2.5	3.0	3.5	4.0	4.5	5.0
F	4	36	60	90	70	40	10

b) Obtain the regression lines for the following data [06]

X	2	3	5	7	9	10	12	15
Y	2	5	8	10	12	14	15	16

Find estimate value of i)when x=0, ii) when y=20

c) A manufacturer of cotter pins knows that, 2 % of the product is defective. If he sells cotter pin boxes of 100 pins and guarantees that not more than 5 pins will be defective in a box, find the approximate probability that a box will fail to meet the guaranteed quality. [05]

OR

Q8) a) A problem on computer mathematics is given to the three students A, B and C. whose chances of solving it are ½, ¾ and 1/3 respectively? What are the probability that the problem will be solved? [05]

b) A random sampling of 200 screws is drawn from the population which represents the size of a screw. If a sample is distributed normally with a mean 3.15 cm and standard deviation 0.025 cm, find expected number of screws whose size falls between 3.12cm and 3.2 cm [05]

c) Calculate the correlation for the following weights (in kgs) of husband (x) and wife (y) [07]

X	65	66	67	67	68	69	70	72
Y	55	58	72	55	66	71	70	50

Q 9) a) Find the angle between the tangent to the curve: [05]

$$\bar{r} = (t^3+2)\bar{i} + (4t-5)\bar{j} + (2t^2-6t)\bar{k} \text{ at t=0 and t=2}$$

b) Find the directional derivative of the function $\phi = e^{2x-y-z}$ at (1,1,1) in the tangent of to the curve $x = e^{-t}$, y=2sint+1, z=t-cost at t=0 [06]

c) Verify whether the following vector field is irrational, if so, find corresponding potential ϕ, [05]

$$\overline{F} = (y\ \sin\ z - \sin x)\ \overline{j} + (x\ \sin\ z - 2yz)\ \overline{j} + (xy\ \cos z + y^2)\ \overline{k}$$

OR

Q 10) a) With usual notations establish the following identities **(any two)** [08]

i) $\nabla x\left(\dfrac{\overline{a}\ x\ \overline{r}}{r^3}\right) = -\dfrac{\overline{a}}{r^3} + \dfrac{3(\overline{a}.\overline{r})}{r^5}\overline{r}$

ii) $\overline{a}\ .\ \nabla\left(\overline{b}.\nabla(\frac{1}{r})\right) = \dfrac{3(\overline{a}.\overline{r})}{r^3}\dfrac{3(\overline{b}.\overline{r})}{.} - \dfrac{(\overline{a}.\overline{b})}{r^3}$

iii) $\nabla.\left[r\nabla\left(\dfrac{1}{r^4}\right)\right] = \dfrac{8}{r^5}$

b) Show that, $\overline{F} = f(r)\overline{r}$ is irrotational . Find $f(r)$ such that $\overline{F}$ is Solenoidal. [4]

c) If the directional derivatives of ϕ = axy+byz+czx at (1,1,1), has maximum magnitude 4 in the direction parallel to X axis, find the values of a,b,c [4]

Q 11 a) Find $\int \overline{F}.d\overline{r}$ for $\overline{F} = (2y+3)\overline{i} + xz\ \overline{j} + (yz+x)\overline{k}$ along the curve $x = 2t^2$, $y = t$, $z = t^3$ from t=0 to t=1 [5]

b) Show that, $\iiint_v \dfrac{dv}{r^2} = \iint_s \dfrac{\overline{r}.\hat{n}}{r^2} dS$ [6]

c) Evaluate $\iint_s (\nabla\ x\ \overline{F}).\overline{ds}$, where $\overline{F} = (x^3 - y^3)\overline{i} - xyz\ \overline{j} + y^3\overline{k}$ and S is the surface $x^2 + 4y^2 + z^2 - 2x = 4$ above the plane x =0 [6]

OR

Q 12 a) Evaluate $\iint_s (x^3\overline{i} + y^3\overline{j} + z^3\overline{k}).\overline{ds}$ where S is surface of sphere. [6]

$x^2 + y^2 + z^2 = 16$

b) Evaluate using Stokes theorem [6]

$\oint (ydx + zdy + xdz)$, where C is the intersection of $x^2 + y^2 + z^2 = a^2$, $x+z = 2a$

c) Show that, the velocity potential $\Phi = \dfrac{1}{2}a(x^2 + y^2 - 2z^2)$ satisfies the Laplce equation. Also determine the stream lines. [5]