Total No of Questions: [12] **SEAT NO. :**

[Total No. of Pages : 4]

S.E. MECH/PROD /AUTO

ENGINEERING MATHEMATICS - III

2008 PATTERN (Semester - I)

Time: 3 Hours **Max. Marks : 100**

Instructions to the candidates:

1) *Answers to the two sections should be written in separate answer books.*
2) *Answer any three questions from each section.*
3) *Neat diagrams must be drawn wherever necessary.*
4) *Figures to the right side indicate full marks.*
5) *Use of Calculator is allowed.*
6) *Assume Suitable data if necessary*

SECTION I

Q1) a) Solve any three [12]

i) $(D^2-1)y = e^{-x}\sin^{-x} + \cos^{-x}$

ii) $(D^2-4D)y = 2\cosh 2x$

iii) $(D^3-3D^2+3D-1)y = e^{2x}\cosh x$

iv) $(\frac{d^2y}{dx^2}+y) = \operatorname{cosec} x$ (by the method of variation of parameter; where $D = \frac{d}{dx}$

b) Solve : $\frac{dx}{dt}+y = \sin t$; $\frac{dy}{dt}+4x = \cos t$ [05]

OR

Q2) a) Solve any three [12]

i) $(D^2-3D+2)y = (x^3+x+1)$

ii) $(D^2+2D+1)y = xe^{-x}\cos x$

iii) $\frac{dx}{y} = \frac{dy}{-x} = \frac{dz}{2x-3y}$

iv) $(2x+1)^2\frac{d^2y}{dx^2} + 2(2x+1)\frac{dy}{dx} + 4y = 4\sin[\log(2x+1)^2]$

b) A 3N weight stretches a spring 15 cms. If the weight is pulled 10 cms below the equilibrium position and released, find the displacement function at any time t [05]

Q3) a) Find Laplace Transform of the following ...... (Any two) [06]

i) $F(t) = \begin{cases}(t-1); & t>1\\ 0; & t<1\end{cases}$

ii) $\int_0^t t\,e^{-4t}\sin 3t\,dt$

iii) $\frac{d}{dt}(\frac{\sin t}{t})$

b) Solve the following equation by using Laplace Transform [06]

$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = te^{-t}$. Given , y(0) = 1, y'(0) = - 2

c) Find the Fourier transform of

$F(x) = \begin{cases} -3, & |x| < 1 \\ x, & |x| > 1 \end{cases}$. Hence evaluate $\int_0^\infty \frac{\sin\lambda}{\lambda}\, d\lambda$. Also deduce that $\int_0^\infty \frac{\sin\lambda}{\lambda}\, d\lambda$ [06]

OR

Q4) a) Find Inverse Laplace Transform (**Any two**) [08]

i) $\dfrac{2s+1}{(s^2+s+1)^2}$

ii) $\dfrac{1}{s}\log(\dfrac{s^2+a^2}{s^2+l^2}$

iii) $\dfrac{1}{s^2(s+1)}$

b) Find Fourier Sine transform of $\frac{e^{-ax}}{x}$ [04]

c) Solve the integral equation $\int_0^\infty f(x)\sin\lambda x\, dx = \begin{cases} 2-x, & 0 \le \lambda \le 2 \\ 0, & \lambda > 2 \end{cases}$ [05]

Q5) a) If $\frac{\partial^2 y}{\partial t^2} = C^2 \frac{\partial^2 y}{\partial x^2}$ represents the vibrations of a string of length l fixed at both ends, find the solution with boundary conditions, [08]

i) y (t, 0) = 0; $\forall\, t$

ii) ii) y (l, t) = 0 ; $\forall\, t$

iii) $(\frac{\partial y}{\partial t})_{t=0} = 0;\ \forall\, t$

iv) Y (x, 0) = k ($lx - x^2$) , for $\forall\, t$, $0 \le x \le l$

b) Solve : $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$ if [08]

i) u is finite for all t

ii) u (0, t) = 0 ; $\forall\, t$

iii) u (l, t) = 0 ; $\forall\, t$

iv) u (x, 0) = u_0 , for $0 \le x \le l$ where l being the length of the bar

OR

Q6) a) An infinitely long uniform metal plate is enclosed between the lines x = 0 and x = 1 for y > 0. The temperature is being zero along the edges x = 0, x = 1 and at y = ∞ . If the edge y = 0 is kept at a constant temperature x (1-x). Find the temperature distribution u(x,y) [08]

.

b) Solve using Fourier transform : [08]

$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$; $0 < x < \infty$, $t > 0$

Subject to the boundary condition

U(0, t) = f(t) ; t > 0 and the initial condition u(x, 0) = 0, x>0

SECTION II

Q7) a) The first four moments about the point 3.5 of a distribution are 0.058, 0.452, 0.0823, and 0.05. Calculate the moments about the mean. Also calculate co-efficient of kurtosis and skewness [06]

b) Given, n=6, $\sum(x_i - 18.5) = -3$; $\sum(y_i - 50) = 20$; $\sum(x_i - 18.5)^2 = 19$; $\sum(y_i - 50)^2 = 850$; $\sum(x_i - 18.5)(y_i - 50) = -120$. Calculate the coefficient of correlation. [05]

c) In a quality control department of tube manufacturing factory, 10 rubber tubes are randomly selected from each day's production for inspection. If not more than 1 of the 10 tubes is found to be defective, the production lot is approved, and otherwise it is rejected. Find the probability of the rejection of a day's production lot if the true proportion of defectives in the lot is 0.3 [05]

OR

Q8) a) In a certain factory turning out razor blades. There is a small chance of $\frac{1}{500}$ for. Use Poisson distribution to calculate the approximate number of packets containing no defective and two defective blades in a consignment of 10, any blade to be defective. The blades are supplied in a packet of 10,000 packets [05]

b) Assuming that the diameter of 1000 brass plugs then consecutively from machine, form a normal distribution mean 0.7515 cm and standard deviation 0.002 cm. How many of the plugs are likely to be approved if the acceptable diameter is 0.752 $\pm$ 0.004 cm. [06]

Given: A(2.25) = 0.4878

A(1.75) = 0.4599

c) Calculate the standard deviation for the following frequency distribution. Decide whether arithmetic mean is good average ? [05]

Wages in Rupees earned per day	0-10	10-20	20-30	30-40	40-50	50-60
Number of labors	5	9	15	12	10	3

Q9) a) If $\overline{r}.\frac{\overline{r}}{dt} = 0$ then prove that $\overline{r}$ have constant magnitude. [04]

b) Find the directional derivate of $\phi = x^2\,yz + 4xz^2$ at the point (1,-2,-1) in the direction of vector $2\overline{i}$ - $\overline{j}$ - $2\overline{k}$ [05]

c) Prove that [08]

i) $\nabla^2(r^2 \log r = \frac{\sigma}{r^2})$

ii) If $\overline{u}$ and $\overline{v}$ *are irrotional vectors then Prove that* $\overline{u}$ and $\overline{v}$ are solinoidal.

OR

Q10) a) Show that $\overline{F} = (y\sin z - \sin x)\overline{i} + (x \sin z + 2yz)\overline{j} + (xy \cos z + y^2)\overline{k}$ is irrotional . Find the scalar potential ϕ such that $\overline{F} = \nabla\phi$. [06]

b) If the directional derivative of ϕ = axy +byz+czx at (1,1,1) as maximum magnitude 8 in a direction parallel to y –axis , find the values of a,b,c. [05]

c) A curve is given by the equation : x= $t^2 + 15$; $y = 4t - 3$; $z = 2t^3 - 6t$. Find the angle between tangents at t= 2 and t=3. [06]

Q11) a) Find the work done in moving a particle from (0,0,0) to (1,1,1) in a force field [05]

$\overline{F} = yz\,\overline{i} + xz\,\overline{j} + xy\overline{k}$

b) Evaluate $\oint_c \overline{F}.d\overline{r}$ by using Stoke's theorem for the surface of paraboliod z= 9-(x^2+y^2) above the plane z=0; where , $\overline{F} = (x^2 + y - 4)\overline{i} + 3xy\overline{j} + (2yz + z^2)\overline{k}$ [06]

c) Use Divergence's theorem to evaluate $\oiint_S \overline{F}.\hat{n}\,ds$ where S is the closed surface of the cylinder $x^2+y^2 = 4\,; z = 0, z = 3$ where, $\overline{F} = 4x\overline{i} - 2y^2\overline{j} + x^2\,\overline{k}$ [06]

OR

Q12) a) Evaluate $\int_c (\sin y - y^3)dx + (xy^2 + x\cos y)dy$ by Green's theorem where C is the circle $x^2+y^2 = a^2$ [05]

b) By using divergence theorem, evaluate $\iint_s (x^2y^3\overline{i} + z^2x^3\overline{j} + x^2y^3)\overline{k}).\overline{ds}$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ [06]

c) Evaluate $\iint_s (\nabla \times \overline{F}).d\overline{s}$ where $\overline{F} = (x^3 - y^3)\overline{i} - xyz\overline{j} + y^3\overline{k}$ and S is the surface $x^2 + 4y^2 + z^2 - 2x = 4$ above the plane x= 0. [06]