

**S.E. CIVIL ENGINEERING**  
**Engineering Mathematics - III**  
**(COURSE - 2012)**

**Time: 2 Hours**

**Max. Marks : 50**

**Instructions to the candidates:**

- 1) Answers Q1 or Q2, Q3 or Q4, Q5 or Q6 and Q7 or Q8.
- 2) Neat diagrams must be drawn wherever necessary.
- 3) Figures to the right side indicate full marks.
- 4) Use of Non programmable calculator is allowed.
- 5) Assume Suitable data if necessary.

Q1) a) Solve any two : [08]

i)  $(D^3 - 3D^2 + 4D - 2)y = e^x + \cos x$

ii)  $(D^2 - 4D + 4)y = 8x^2 e^{2x} \sin 2x$

iii)  $[(x+1)^2 D^2 + (X+1)D]y = (2x+3)(2X+4)$   $D = \frac{d}{dx}$

b) Solve  $28x + 4y - z = 32$  [04]

$2x + 17y + 4z = 35$

$x + 3y + 10z = 24$  by Gauss Seidel method

**OR**

Q2) a) A light horizontal strut AB of length L is freely pinned at A & B is under the [04]

action of equal and opposite compressive forces P at each of its ends and carries a load W at its centre. Find the deflection.

[Hint :  $EI \frac{d^2y}{dx^2} = -\frac{W}{2}x - Py$ ,  $x = 0$ ,  $y = 0$ ,  $x = \frac{L}{2}$ ,  $\frac{dy}{dx} = 0$ ,  $\frac{P}{EI} = n^2$ ]

b) Apply fourth order Runge - Kutta method to  $\frac{dy}{dx} = 3x + \frac{y}{2}$ ,  $y(0) = 1$  [04]

to determine y (0.1) taking h = 0.1

c) Solve  $4X_1 + 2X_2 + 14X_3 = 14$  [04]

$2X_1 + 17X_2 - 5X_3 = -101$

$14X_1 - 5X_2 + 83X_3 = 155$  by Cholesky's method

Q3) a) Find the coefficient of correlation of the following table [04]

x	5	7	9	11
y	8	4	16	12

b) If the probability that an individual suffers a bad reaction from a certain injection [04]

is 0.002, determine the probability that out of 1000 individuals, more than two individuals will suffer a bad reaction.

- c) Find the directional derivative of  $\Phi = xy^2 + yz^3$  at  $(2, 1, 1)$  in the direction normal to the surface  $x^2y + y^2x + yz^2 = 3$  at  $(1, 1, 1)$ . [04]

OR

- Q4) a) Calculate first three moments about mean for the following data [04]

x	61	64	67	70	73
f	5	18	42	27	8

- b) Show that (any one) [04]

i)  $\nabla \cdot \left( r \nabla \frac{1}{r^3} \right) = \frac{3}{r^4}$

ii)  $\vec{F} = (x + 3y)\mathbf{i} + (y - 2z)\mathbf{j} + (az + x)\mathbf{k}$  is solenoidal. Find value of  $a$ .

- c)  $\vec{F} = (y e^x + e^z)\mathbf{i} + (z e^y + e^x)\mathbf{j} + (x e^z + e^y)\mathbf{k}$  is irrotational and find their scalar potential [04]

- Q5) a) Find the work done by the force  $\vec{F} = 3x^2\mathbf{i} + (2xz - y)\mathbf{j} + 2\mathbf{k}$  along the curve  $x^2 = 4y, 3x^3 = 8z$  from  $x=0$  to  $x=z$ . [04]

- b) Apply Stokes theorem to calculate  $\int_c 4y dx + 2z dy + 6y dz$ , where  $c$  is the curve at intersection of  $x^2 + y^2 + z^2 = 6z$  &  $z = (x+3)$  [05]

- c) Use divergence theorem to evaluate  $\iint_s (x dy dz + y dz dx + z dx dy)$  over the surface of a sphere of radius  $a$ . [04]

OR

- Q6) a) Use Green's theorem to evaluate [04]

$\int_c \vec{F} \cdot d\vec{r}$  where  $\vec{F} = y^3\mathbf{i} - x^3\mathbf{j}$  and 'c' is the circle  $x^2 + y^2 = a^2, z=0$

- b) Use Divergence theorem to evaluate  $\iiint_s \vec{F} \cdot d\vec{s}$  where  $\vec{F} = 4x\mathbf{i} - 2y^2\mathbf{j} + z^2\mathbf{k}$  and  $s$  is surface bounding the region  $x^2 + y^2 = 4, z = 0$  and  $z = 3$ . [05]

- c) Evaluate  $\iint_s (\nabla * \vec{F}) \cdot \hat{n} ds$  where  $s$  is the curved surface of the paraboloid  $x^2 + y^2 = 2z$ , bounded by the plane  $z = 2$ , where  $\vec{F} = 3(x - y)\mathbf{i} + 2xz\mathbf{j} + xy\mathbf{k}$  [04]

- Q7) a) A string of length  $L$  is stretched and fastened to two ends. Motion is started by displacing the string in the form  $u(x) = a \sin\left(\frac{\pi x}{l}\right)$  from which it is released at  $t=0$ . [07]

Find the displacement 'u' at any time 't', if it satisfies the equation  $\frac{\partial^2 y}{\partial t^2} = e^2 \frac{\partial^2 y}{\partial x^2}$

- b) Solve  $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$  if [06]

i)  $u(x, \infty)$  is finite

ii)  $u(0, t) = 0$

iii)  $u(l, t) = 0$

$$\text{iv) } u(x, 0) = x, \quad 0 < x < l$$

OR

- Q8) a) An infinitely long plane uniform plate is bounded by two parallel edges in the  $y$  direction and an end at right angles to them. The breadth of the plate is  $\pi$ . The end is maintained at temperature  $u_0$  at all points and other edges at zero temperature. Find steady state temperature  $u(x, y)$ . if it satisfies  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$  [07]

- b) Solve  $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$  if [06]

i)  $u(x, t)$  is bounded

ii)  $u(0, t) = 0$

iii)  $u(l, t) = 0$

iv)  $u(x, 0) = \frac{u_0 x}{l}, \quad 0 < x < l$