Total No of Questions: [08] **SEAT NO. :**

[Total No. of Pages : 03]

S.E. 2012 (MECH/PROD)
Engineering Mathematics III

Time: 2 Hours **Max. Marks : 50**

Instructions to the candidates:

1) *Attempt Q. 1 or Q. 2, Q. 3 or Q.4, Q. 5 or Q.6, Q. 7 or Q.8,*
2) *Neat diagrams must be drawn wherever necessary.*
3) *Figures to the right side indicate full marks.*
4) *Use of Calculator is allowed.*
5) *Assume Suitable data if necessary*

SECTION I

Q1) a) Solve any Two of the following: [08]

i) $(D^2 + D + 1)\,y = x \sin x$

ii) $(D^2 - 4D + 4)\,y = e^x \cos^2 x$

iii) $(3x+2)^2 \frac{d^2y}{dx^2} + 3\,(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$

b) Find the Fourier Transform of: [04]

$$f(x) = 1, \quad |x| \le a$$
$$= 0, \quad |x| > a$$

Q2) a) A body weighing W = 20 N is hung from the spring. A pull of 40 N will stretch the spring to 10 cm. The body is pulled down to 20 cm bellow the static equilibrium position and then released. Find the displacement of the body from its equilibrium position in time 't' seconds. Also find the maximum velocity and Period of oscillation. [04]

b) Solve any one of the following: [04]

i) Find the Laplace Transform of: $f(x) = t\,e^{3t}\,\sin 2t$

ii) Find inverse Laplace Transform of:

$$F(s) = \frac{1}{(S-2)^4\,(S+3)}$$

c) Solve the following Differential equation by Laplace Transform method. [04]

$$\frac{d^2y}{dt^2} + 2\,\frac{dy}{dt} + 5y = e^{-t}\sin t$$

Given that : $y(0) = 0\,,\ y'(0) = 1$

Q3) a) Following are the values of import of raw material and export of finished product in suitable units [04]

Export	10	11	14	14	20	22	16	12	15	13
Import	12	14	15	16	21	26	21	15	16	14

Calculate the coefficient of correlation between the import and export values.

b) Find curl curl $\overline{F}$ at the point (1, 1, 2) where $\overline{F} = x^2y\,\overline{i} + x\,y\,z\,\overline{j} + z^2y\,\overline{k}$ [04]

c) Prove the following (any one)

i) $\nabla \cdot \left(\frac{\overline{a} \times \overline{r}}{r}\right) = 0$

ii) $\overline{a} \cdot \nabla \left[\overline{b} \cdot \nabla \left(\frac{1}{r}\right)\right] = \frac{3\,(\overline{a} \cdot \overline{r})(\overline{b} \cdot \overline{r})}{x^5} - \frac{(\overline{a} \cdot \overline{b})}{x^3}$

Q4) a) In a distribution, exactly normal, 7 % of the items are under 35 and 89 % are under 63. Find the mean and standard deviation of the distribution. [04]
[$A_1 = 0.43$, $z_1 = 1.48$, $A_2 = 0.39$, $z_2 = 1.23$]

b) Number of road accidents on a highway during a month follows a Poisson distribution with mean 5, Find the probability that in a certain month, number of accidents on the highway will be: [04]

i) Less than 3
ii) Between 3 and 5

c) Find the constants a and b, so that the surface $ax^2 - byz = (a+2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point (1, -1, 2) [04]

Q5) a) Find the work done in moving a particle along the path $\boldsymbol{x = 2\,t^2}$, $\boldsymbol{= t}$, $\boldsymbol{z = t^3}$, from t = 0 to t = 1 in a force field $\overline{\boldsymbol{F}} = (\boldsymbol{2\,y+3})\,\overline{\boldsymbol{i}} + \boldsymbol{x\,z}\,\overline{\boldsymbol{j}} + (\boldsymbol{y\,z - x})\,\overline{\boldsymbol{k}}$ [04]

b) Evaluate: $\iint_S (x\,\overline{i} + y\,\overline{j} + z^2\,\overline{k}) \cdot d\overline{S}$, where s is curved surface of cylinder $x^2 + y^2 = 4$ bounded by the planes z = 0 and z = 2. [05]

c) Apply Stokes's theorem to calculate $\int_C (y\,dx + z\,dy + x\,dz)$, C being intersection of $x^2 + y^2 + z^2 = a^2$, $x + z = a$ [04]

Q6) a) i) Sing Green's lemma evaluate [04]

$\oint_C x^2dx + x\,y\,dy$, where C is the boundary of region R which is enclosed by $y = x^2$, and $y = x$

b) Evaluate $\iint_S (\nabla \times \overline{F}) \cdot \hat{n}\,dS$, where S is the curved surface of the paraboloid $x^2 + y^2 = 2z$ bounded by the plane z = 2 and $\overline{F} = 3(x-y)\,\overline{i} + 2xz\,\overline{j} + xy\,\overline{k}$ [05]

c) Prove that: [04]

$\iint_S (\phi\nabla\psi - \psi\nabla\phi) \cdot d\overline{S} = \iiint_v (\phi\nabla^2\psi - \psi\nabla^2\phi)\,dv$, where S is closed surface enclosing volume V

Q7) a) A string of length l is stretched and fastened to two ends. Motion is started by displacing the string in the form $u(x) = a\,\sin\left(\frac{\pi x}{l}\right)$ from which it is released at t = 0, Find the displacement u at any time 't', if it satisfies the equation [07]

$$\frac{\partial^2 y}{\partial t^2} = c^2\,\frac{\partial^2 y}{\partial x^2}$$

b) Solve: $\frac{\partial u}{\partial t} = a^2\frac{\partial^2 u}{\partial x^2}$ if [06]

i) $u(x, \infty)$ is finite
ii) $u(0, t) = 0$
iii) $u(l, t) = 0$
iv) $u(x, 0) = x, \quad 0 < x < l$

Q8) a) An infinitely long plane uniform plate is bounded by two parallel edges in the y direction and an end at right angles to them. The breadth of the plate is π. The end is maintained at temperature u_0 at all points and other edges at zero temperature. Find steady state temperature u(x,y), if it satisfies $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ [07]

b) Use Fourier Transform to solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$; $0 < x < \infty,\ t > 0$, where $u(x,t)$ Satisfies the conditions: [06]

i) $u(x,t) < M$

ii) $\left(\frac{\partial u}{\partial t}\right)_{x=0} = 0$ at t >0

iii) $u(x,0) = x, \quad 0 < x < 1$
$ = 0, \quad x > 1$