Total No. of Questions—**12**] [Total No. of Printed Pages—**7**

Seat No.	

[4856]-11

F.E. (First Semester) EXAMINATION, 2015

ENGINEERING MATHEMATICS—I

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

***N.B.* :—** (*i*) Answer *three* questions from Section I and *three* questions from Section II.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.

(*v*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Reduce the matrix : [5]

$$A = \begin{bmatrix} 1 & 2 & 3 & -2 \\ 2 & -2 & 1 & 3 \\ 3 & 0 & 4 & 1 \end{bmatrix}$$

to echelon form and determine its rank.

(*b*) Discuss consistency and solve if consistent : [6]

$$2x_1 - x_2 + 3x_3 = 1$$
$$3x_1 + 2x_2 + x_3 = 3$$
$$x_1 - 4x_2 + 5x_3 = -1.$$

(*c*) Verify Caley-Hamilton's theorem for the matrix : [6]

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 1 \\ 1 & -1 & 0 \end{bmatrix}.$$

Or

2. (*a*) Find eigenvalues and eigenvectors for the matrix : [6]

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

(*b*) Examine the vectors : [5]

$X_1 = (2, 2, 1)^T$, $X_2 = (1, 3, 1)^T$,

$X_3 = (1, 2, 2)$

for linear dependence or independence.

(*c*) Is the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$ orthogonal ? If not, can it be converted to orthogonal matrix. [6]

3. (*a*) If $\dfrac{z-1}{z+i}$ is purely imaginary, show that its locus is circle. [5]

(*b*) Solve by using De-Moivre's theorem $x^5 - 1 = 0$. [5]

(*c*) If α, β are the roots of the equation $x^2 - 2x + 2 = 0$, then show that $\alpha^n + \beta^n = 2^{n/2} \,.\, 2 \cos \dfrac{n\pi}{4}$. [6]

Or

4. (*a*) Separate real and imaginary parts of $\cos^{-1}\left(\dfrac{3i}{4}\right)$. [5]

(*b*) Show that : [5]

$$\tanh\left(\log \sqrt{3}\right) = \frac{1}{2}.$$

(*c*) By considering principal values only express $\left(1 + i\sqrt{3}\right)^{1+i\sqrt{3}}$ in A + *i*B form. [6]

5. (*a*) If $y = e^x (\sin x + \cos x)$, then find y_n. [5]

(*b*) If $y = (x^2 - 1)^n$, then show that : [6]

$$(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0.$$

(*c*) Discuss convergence or divergence (any *one*) : [6]

(*i*) $\displaystyle\sum_{n=1}^{\infty} \frac{5^n + a}{3^n + b} \quad (a > 0,\ b > 0)$

(*ii*) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{1}{n}\right)$.

Or

6. (*a*) If $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then show that [5]

$$y_n = 2(-1)^{n-1}(n-1)!\sin^n \phi \sin n\phi$$

where $\phi = \tan^{-1}\left(\frac{1}{x}\right)$.

(*b*) If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, then show that [6]

$$(1-x^2)y_{n+1} - (2n+1)xy_n - n^2 y_{n-1} = 0.$$

(*c*) Attempt (any *one*) : [6]

(*i*) Test absolute or conditional convergence of :

$$1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} + \ldots\ldots\ldots$$

(*ii*) Find range of convergence for :

$$\sum_{n=1}^{\infty} \frac{x^n}{2^n}\ (x > 0).$$

SECTION II

7. (*a*) Expand $\log(1 + x + x^2 + x^3)$ upto x^6. [6]

(*b*) Obtain expansion of $\tan^{-1} x$ in the powers of '$x - 1$'. [5]

(*c*) Attempt (any *one*) : [6]

(*i*) Evaluate :

$$\lim_{x\to 0} \frac{xe^x - \log(1+x)}{x^2}.$$

(*ii*) Evaluate :

$$\lim_{x\to 0}\left(\frac{a}{x} - \cot\frac{x}{a}\right).$$

Or

8. (*a*) Expand $x^3 + 7x^2 + x - 6$ in the powers of '$x - 3$'. [6]

(*b*) Show that : [5]

$$\log \sec x = \frac{x^2}{2} + \frac{x^4}{12} + \frac{x^6}{45} + \ldots\ldots\ldots\ldots$$

(*c*) Attempt (any *one*) : [6]

(*i*) Find a, b so that :

$$\lim_{x \to 0} \frac{a \sin 2x - b \tan x}{x^3} = 1.$$

(*ii*) Evaluate :

$$\lim_{x \to 0} \left(\frac{\tan x}{x} \right)^{\frac{1}{x^2}}.$$

9. Attempt any *two* :

(*a*) If $z = e^{ax + by} f(ax - by)$, then show that : [8]

$$b \frac{\partial z}{\partial x} + a \frac{\partial z}{\partial y} = 2\, abz.$$

(*b*) If $x = \frac{r}{2}\left(e^{\theta} + e^{-\theta}\right)$, $y = \frac{r}{2}\left(e^{\theta} - e^{-\theta}\right)$, then show that : [8]

$$\left(\frac{\partial x}{\partial r}\right)_{\theta} = \left(\frac{\partial r}{\partial x}\right)_{y}.$$

(*c*) If $u = f(x^2 - y^2, y^2 - z^2, z^2 - x^2)$, then show that : [8]

$$\frac{1}{x}\frac{\partial u}{\partial x} + \frac{1}{y}\frac{\partial u}{\partial y} + \frac{1}{z}\frac{\partial u}{\partial z} = 0.$$

Or

10. Attempt (any *two*) :

(*a*) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, then show that : [8]

$$x^2 u_{xx} + 2xy\, u_{xy} + y^2 u_{yy} = \sin 2u\,[2\cos 2u - 1].$$

(*b*) If : [8]

$$ax^2 + by^2 + cz^2 = 1,\ lx + my + nz = 0$$

then find :

$$\frac{dy}{dz}.$$

(*c*) If $x = e^u \tan v,\ y = e^u \sec v$, then find : [8]

$$\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right)\left(x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y}\right).$$

11. (*a*) In calculating volume of right circular cylinder, errors of 2% and 1% are made in measuring height and radius respectively find percentage error in calculated volume. [5]

(*b*) Examine the functions : [6]

$$u = \frac{x - y}{x + y},\ v = \frac{x + y}{x}$$

for functional dependence and find relation if they are functionally dependent.

(*c*) Discuss maximum or minimum values of the function $3x^2 - y^2 + x^3$. [6]

Or

12. (*a*) Verify $JJ^1 = 1$ if $x = u(1 - v)$, $y = uv$. [5]

(*b*) Find the points on the surface $z^2 = xy + 1$, which is at minimum distance from origin. [6]

(*c*) If $x^2 + y^2 + u^2 - v^2 = 0$, $uv + xy = 0$, then show that : [6]

$$\frac{\partial(u, v)}{\partial(x, y)} = \frac{x^2 - y^2}{u^2 + v^2}.$$