Total No. of Questions—**12**] [Total No. of Printed Pages—**8**

Seat No.	

[4856]-21

F.E. (II Sem.) EXAMINATION, 2015

ENGINEERING MATHEMATICS—II

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

***N.B.* :—** (*i*) Section I : Solve Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**.

Section II : Solve Q. No. **7** *or* Q. No. **8**, Q. No. **9** *or* Q. No. **10**, Q. No. **11** *or* Q. No. **12**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of non-programmable electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Form the differential equation whose general solution is : [6]

$$y = a\cos(\log x) + b\sin(\log x).$$

(*b*) Solve any *two* : [10]

(*i*) $(2x + 3y + 4)dx - (4x + 6y + 5)dy = 0$

(*ii*) $(x+1)\frac{dy}{dx} - y = e^{5x}(x+1)^2$

(*iii*) $(y\log y)dx + (x - \log y)dy = 0.$

Or

2. (*a*) Form the differential equation whose general solution is : [6]

$$y = ax + \frac{b}{x}.$$

(*b*) Solve any *two* : [10]

(*i*) $(2xy)dy = (3y^2 + x^2)dx$

(*ii*) $x\frac{dy}{dx} + y = x^3y^6$

(*iii*) $\frac{dy}{dx} = (4x + y + 1)^2.$

3. Solve any *three* : [18]

(*i*) Find the orthogonal trajectories of the curve given by :

$$r = a(1 + \cos\theta)$$

(*ii*) A pipe 20 cm in diameter steam at 150°C and is protected with covering 5 cm thick for which K = 0.0025. If the temperature of the outer surface of the covering is 40°C. Find the temperature half way through the covering.

(*iii*) A constant e.m.f. 500 volts is applied to a circuit containing a resistance 250 ohms, an inductance of 640 henry. If the initial current is zero, find the time that elapses before it reaches 80% of its maximum value.

(*iv*) A body originally at 60°C cools down to 40°C in 20 minutes, the temperature of the air being 36°C. What will be the temperature of the body after 30 minutes from the original ?

Or

4. Solve any *three* : [18]

(*i*) A particle is moving in a straightline with an acceleration kx directed towards origin. If it starts from rest at a distance a from the origin, prove that it will arrive at origin at the end of time $\frac{\pi}{2\sqrt{k}}$.

(*ii*) A body at temperature 100°C is placed in a room whose temperature is 20°C and cools to 60°C in 5 minutes. Find the time at which the temperature of the body drops to 50°C.

(*iii*) An electrical circuit contains an inductance of 5 henries and a resistance of 12 ohms in series with an e.m.f. $120 \sin (20t)$ volts. Find the current at $t = 0.01$, if it is zero when $t = 0$.

(*iv*) Find the orthogonal trajectories of the curves given by $x^2 + 2y^2 = c^2$.

5. (*a*) Find the Fourier series expansion for $f(x)$, if : [9]

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

Hence deduce that :

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots = \frac{\pi^2}{8}.$$

(*b*) If [7]

$$u_n = \int_0^{\frac{\pi}{4}} \tan^n \theta \, d\theta,$$

then show that $n\left(u_{n+1} + u_{n-1}\right) = 1$ and hence find :

$$\int_0^{\frac{\pi}{4}} \tan^4 \theta \, d\theta.$$

Or

6. (*a*) Obtain the first three coefficients in the Fourier cosine series for y, where y is given in the following table : [8]

x	y
0	4
1	8
2	15
3	7
4	6
5	2

(*b*) Evaluate : [4]

$$\int_0^{\infty} \frac{x^5}{5^x} \, dx.$$

(*c*) Evaluate : [4]

$$\int_2^5 (x-2)^{\frac{1}{3}} (5-x)^{\frac{1}{3}} \, dx.$$

SECTION II

7. (*a*) Trace the following curves (any *two*) : [8]

(*i*) $x(x^2 + y^2) = a(x^2 - y^2), \ a > 0$

(*ii*) $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$

(*iii*) $r = a \sin 3\theta$.

(*b*) Evaluate : [4]

$$\int_0^1 \frac{x^a - 1}{\log x}\, dx,$$

using DUIS.

(*c*) Find the length of upper arc of one loop of lemniscate $r^2 = a^2 \cos 2\theta$. [5]

Or

8. (*a*) Trace the following curves (any *two*) : [8]

(*i*) $x^2y^2 = a^2(y^2 - x^2)$

(*ii*) $x = a(t + \sin t) \ y = a(1 + \cos t)$

(*iii*) $r = a(1 + 2\cos\theta)$.

(*b*) Show that : [5]

$$\frac{d}{dx}\left[erf\left(x\right)\right] = \frac{2}{\sqrt{x}} e^{-x^2}$$

and use it to show that :

$$\frac{d}{dx}\left(erf\left(ax^n\right)\right) = \frac{2an}{\sqrt{x}} x^{n-1} e^{-a^2x^{2n}}.$$

(*c*) Show that length of arc of curve $ay^2 = x^3$ from origin to the point whose abcissa '*b*' is : [4]

$$\frac{1}{27\sqrt{a}}(ab + 4a)^{3/2} - \frac{8a}{27}.$$

9. (*a*) Find equation of sphere which passes through the points (1, 0, 0) (0, 2, 0) (0, 0, 3) and has its radius as small as possible. [6]

(*b*) Find the equation of a right circular cone with vertex at origin '*o*' makes equal angle with co-ordinate axes and cone passes through the line drawn from '*o*' with direction cosines proportional to 1, –2, 2. Find the equation of the cone. [6]

(*c*) Find the equation of right circular cylinder whose guiding curve is : [5]

$$x^2 + y^2 + z^2 = 9 \text{ and}$$

$$x - y + z = 3.$$

Or

10. (*a*) Show that the plane : [6]

$$2x - 2y + z + 12 = 0$$

touches the sphere :

$$x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$$

and find the point of contact.

(*b*) Find the equation of a quadratic cone which passes through the three co-ordinate axes and the three mutually perpendicular lines : [6]

$$\frac{x}{1} = \frac{y}{-2} = \frac{z}{3};$$

$$\frac{x}{1} = \frac{y}{-1} = \frac{z}{-1};$$

$$\frac{x}{5} = \frac{y}{4} = \frac{z}{1}.$$

(*c*) Find the equation of right circular cylinder of radius '5' whose axis is the line : [5]

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}.$$

11. Solve (any *two*) :

(*a*) Evaluate : [8]

$$\int_0^1 dx \int_y^\infty e^{-y} \; y^x \; \log y \; dy.$$

(*b*) Find the volume of the solid cut from the sphere : [8]

$$x^2 + y^2 + z^2 = a^2$$

by the cone :

$$x^2 + y^2 = z^2.$$

(*c*) Find Moment of Inertia of one loop of lemniscate $r^2 = a^2 \cos 2\theta$ about initial line. [8]

Or

12. Solve any *two* of the following :

(a) Find the area of loop of the curve : [8]

$$y^2 = \frac{x^2}{(4 - x)(x - 2)}$$

and the two asymptotes.

(b) Evaluate : [8]

$$\iint xyz \, dx \, dy \, dz$$

taken throughout the ellipsoid :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

(c) Find the centroid of the region bounded by $z = 4 - x^2 - y^2$ and xy-plane. [8]