Total No. of Questions—**8**] [Total No. of Printed Pages—**4+1**

Seat No.	

[4856]-101

F.E. EXAMINATION, 2015

ENGINEERING MATHEMATICS—I

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Attempt Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of logarithmic tables electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Examine for consistency of the system of equations : [4]

$$2x - 3y + 5z = 1$$
$$3x + y - z = 2$$
$$x + 4y - 6z = 1$$

if consistent solve it.

(*b*) Find the eigenvalues of matrix : [4]

$$\begin{bmatrix} 1 & 0 & -4 \\ 0 & 5 & 4 \\ -4 & 4 & 3 \end{bmatrix}$$

hence find eigenvector corresponding to highest eigenvalue.

(*c*) Find the complex number z if $\operatorname{amp}(z + 2i) = \frac{\pi}{4}$ and $\operatorname{amp}(z - 2i) = \frac{3\pi}{4}$. [4]

Or

2. (*a*) Examine for the linear dependence or independence. If dependent, find the relation among the following vectors (1, 1, 1), (1, 2, 3), (2, 3, 8). [4]

(*b*) Find all values of $(1 + i)^{1/4}$. [4]

(*c*) If $\sin(\alpha + i\beta) = x + iy$, then prove that : [4]

(*i*) $\frac{x^2}{\cosh^2 \beta} + \frac{y^2}{\sinh^2 \beta} = 1,$

(*ii*) $\frac{x^2}{\sin^2 \alpha} - \frac{y^2}{\cos^2 \alpha} = 1.$

3. (*a*) Test convergence of the series (any *one*) : [4]

(*i*) $\frac{1}{2+3} + \frac{2}{2+3^2} + \frac{3}{2+3^3} + \ldots\ldots\ldots$

(*ii*) $\sum_{n=1}^{\infty} \frac{\sqrt{n+1} + \sqrt{n}}{n^3}.$

(*b*) Prove that : [4]

$$e^{x \cos x} = 1 + x + \frac{x^2}{2} - \frac{x^3}{3} - \ldots\ldots\ldots .$$

(*c*) Find *n*th derivative of [4]

$$\frac{x+1}{(x-1)(x+2)(x-3)}.$$

Or

4. (*a*) Solve any *one* : [4]

(*i*) $\lim_{x \to 0} (\cos x)^{1/x^2}$

(*ii*) $\lim_{x \to 0} \dfrac{e^{ax} - e^{-ax}}{\log(1 + bx)}$.

(*b*) Using Taylor's theorem, expand $x^4 - 5x^3 + 5x^2 + x + 2$ in powers of $x - 2$. [4]

(*c*) If $y = \cos(m \log x)$, then prove that : [4]

$$x^2 y_{n+2} + (2n + 1) x y_{n+1} + (m^2 + n^2) y_n = 0.$$

5. Solve any *two* :

(*a*) If [6]

$$u = \tan(y + ax) + (y - ax)^{3/2},$$

where a is a constant, then show that :

$$\frac{\partial^2 u}{\partial x^2} = a^2 \frac{\partial^2 u}{\partial y^2}.$$

(*b*) If [7]

$$u = x^3 f\left(\frac{y}{x}\right) + \frac{1}{y^3} \phi\left(\frac{x}{y}\right),$$

prove that :

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 9u.$$

(*c*) If [6]

$$u = f(x - y, y - z, z - x),$$

then find the value of :

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}.$$

Or

6. Solve any *two* :

(*a*) If $u = mx + ny$, $v = nx - my$, where m, n are constants, then find the value of : [6]

$$\left(\frac{\partial u}{\partial x}\right)_y \cdot \left(\frac{\partial y}{\partial v}\right)_x \cdot \left(\frac{\partial x}{\partial u}\right)_v \cdot \left(\frac{\partial v}{\partial y}\right)_u .$$

(*b*) If [7]

$$u = \tan^{-1}\left[\frac{x^3 + y^3}{x + y}\right],$$

then prove that :

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \sin 2u\left[1 - 4\sin^2 u\right].$$

(*c*) If [6]

$$z = f(x, y), \ x = \frac{\cos u}{v}, \ y = \frac{\sin u}{v},$$

then prove that :

$$v\frac{\partial z}{\partial v} - \frac{\partial z}{\partial u} = (y - x)\frac{\partial z}{\partial x} - (y + x)\frac{\partial z}{\partial y}.$$

7. (*a*) If [4]

$$u = \frac{y - x}{1 + xy}, \ v = \tan^{-1} y - \tan^{-1} x$$

find $\dfrac{\partial(u, v)}{\partial(x, y)}$.

(*b*) Prove that : [5]

$u = y + z$, $v = x + 2z^2$, $w = x - 4yz - 2y^2$,

are functionally dependent and find relation.

(*c*) As dimensions of a triangle ABC are varied, show that the maximum value of $\cos A \cos B \cos C$ is obtained when the triangle is equilateral. [4]

Or

8. (*a*) If $u + v^2 = x,\ v + w^2 = y,\ w + u^2 = z$ find $\frac{\partial u}{\partial x}$. [4]

(*b*) In estimating the cost of a pile of bricks measured 2 m × 15 m × 1.2 m, the top of the pile is streched 1% beyond the standard length. If the count is 450 bricks in 1 cubic meter and bricks cost Rs. 450 per thousand, find the approximate error in cost. [5]

(*c*) Find the minimum value of $x^2 + y^2$, subject to the condition $ax + by = c$. [4]