Total No. of Questions—**8**] [Total No. of Printed Pages—**4+1**

Seat No.	

[4856]-201

F.E. EXAMINATION, 2015

ENGINEERING MATHEMATICS—II

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

N.B. **:—** (*i*) Attempt *four* questions : Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of electronic non-programmable calculator is allowed.

(*v*) Assume suitable data wherever necessary.

1. (*a*) Solve the following differential equations : [8]

(*i*) $xy\frac{dy}{dx} = \left(1 - x^2\right)\left(1 + y^2\right)$

(*ii*) $\cos y - x\sin y\frac{dy}{dx} = \sec^2 x.$

(*b*) An e.m.f. $200e^{-5t}$ is applied to a series circuit containing of 20 ohm resistor and 0.01 F capacitor. Find the charge and current at any time assuming that there is no initial charge on the capacitor. [4]

P.T.O.

Or

2. (*a*) Solve : [4]

$$2y\,dx + \left(2x \log x - xy\right)dy = 0.$$

(*b*) Solve the following : [8]

(*i*) A body starts moving from rest is opposed by a force per unit mass of value cx and resistance per unit mass of value bv^2, where x and v are the displacement and velocity of the body at that instant. Show that the velocity of the body is given by :

$$v^2 = \frac{c}{2b^2}\left(1 - e^{-2bx}\right) - \frac{cx}{b}.$$

(*ii*) The inner and outer surfaces of a spherical shell are maintained at T_0 and T_1 temperature respectively. If the inner and outer radii of the shell are r_0 and r_1 respectively and thermal conductivity of the shell is k, find the amount of heat loss from the shell per unit time. Find also the temperature distribution through the shell.

3. (*a*) Obtain the first three coefficient in the Fourier cosine series for y using practical harmonic analysis : [5]

x	***y***
0	4
1	8
2	15
3	7
4	6
5	2

(*b*) Evaluate : [3]

$$\int_3^5 (x-3)^{1/2}\,(5-x)^{1/2}\,dx.$$

(*c*) Trace the following curve (any *one*) : [4]

(*i*) $ay^2 = x^2(a - x)$

(*ii*) $r = a(1 + \cos\theta)$.

Or

4. (*a*) If [4]

$$I_n = \int_0^{\pi/2} x^n \cos x\, dx$$

prove that :

$$I_n = \left(\frac{\pi}{2}\right)^n - n(n-1)\, I_{n-2}.$$

(*b*) Show that : [4]

$$\int_0^{\infty} e^{-x^2 - 2bx}\, dx = \frac{\sqrt{\pi}}{2}\, e^{b^2}\left[1 - erf(b)\right].$$

(*c*) Find the arc length of the curve (using rectification) $r = 2a\cos\theta$. [4]

5. (*a*) Find the equation of the sphere which passes through the point (3, 1, 2) and meets X o Y plane in a circle of radius 3 units with centre at (1, –2, 0). [5]

(*b*) Find the equation of right circular cone whose vertex is at the origin with axis $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and has a semi-vertical angle 30°. [4]

(*c*) Find the equation of right circular cylinder of radius 2 whose axis passes through (1, 2, 3) and has direction cosines proportional to 2, 1, 2. [4]

Or

6. (*a*) Find the equation of the sphere passing through the circle $x^2 + y^2 + z^2 = 9$, $2x + 3y + 4z = 5$ and the point (1, 2, 3). [5]

(*b*) Find the equation of right circular cone whose vertex is (1, –1, 1) and axis is parallel to $x = \frac{-y}{2} = -z$ and one of its generators has direction cosines proportional to (2, 2, 1). [4]

(*c*) Find the equation of right circular cylinder of radius 4 with axis passing through origin and making equal angles with the co-ordinate axes. [4]

7. Attempt any *two* of the following :

(*a*) Evaluate : [6]

$$\int_0^1 dx \int_1^\infty e^{-y}\, y^x \log y \, dy.$$

(*b*) Evaluate : [7]

$$\iiint \left(x^2y^2 + y^2z^2 + z^2x^2\right) dx\ dy\ dz$$

throughout the volume of the sphere $x^2 + y^2 + z^2 = a^2$.

(*c*) Find the moment of inertia of one loop of the lemniscate $r^2 = a^2 \cos 2\theta$ about initial line. [6]

Or

8. Attempt any *two* of the following :

(*a*) Evaluate : [7]

$$\int_0^a \int_0^{\sqrt{a^2 - x^2}} \sin\left\{\frac{\pi}{a^2}\left(a^2 - x^2 - y^2\right)\right\} dx\ dy.$$

(*b*) Evaluate : [6]

$$\int_0^\infty \int_0^\infty \int_0^\infty \frac{dx\ dy\ dz}{\left(1 + x^2 + y^2 + z^2\right)^2}.$$

(*c*) Find the C.G. of the loop of the curve : [6]

$$y^2(a + x) = x^2(a - x).$$