Total No. of Questions : 8]

SEAT No. :

P4259

[Total No. of Pages : 3

[4860]-1064

M.E. (Mechanical) (Design Engineering)

ADVANCED MATHEMATICS

COMMON TO NET AUTOMOTIVE CADME

(2013 Credit Pattern) (Semester - I)

Time :3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) ***Answer any five questions.***
2) ***Neat diagrams must be drawn wherever necessary.***
3) ***Figures to the right indicate full marks.***
4) ***Use of logarithmic tables, electronic pocket calculator is allowed.***
5) ***Assume Suitable data if necessary.***

Q1) a) Apply Gram-Schmidt method to the given vectors to get an orthonormal basis.

(1, 2, 1), (–3, –4, –1) and (–4, –7, 0). **[5]**

b) Evaluate $\oint_C \frac{\left(z^2+\cos^2 z\right)}{\left(z-\pi/4\right)^3} dz$ where C is $|z|=1$. **[5]**

Q2) a) If $\omega = \phi + i\psi$ represents the complex potential for an electric field and $\phi = -2xy + \frac{y}{x^2+y^2}$, determine the function ψ. **[5]**

b) Find the Laplace transform of $t^2\, u(t-3) + e^{2t} \cos 3t\, \delta(t-2) + e^{2t}\ erf\ 2\sqrt{t}$. **[5]**

Q3) a) Solve by series method the following equation.

$$\left(1-x^2\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0.$$ **[5]**

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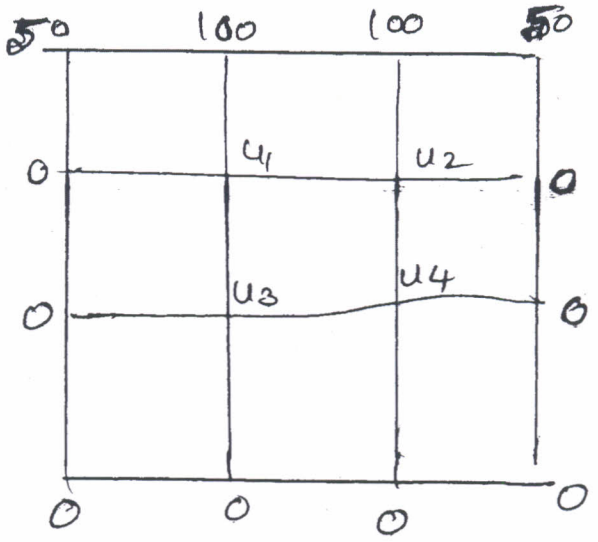
b) Using Laplace transform, find the solution of the initial value problem.

$$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=6e^{-t},\ y(0)=-2$$
$$y'(0)=8$$

[5]

Q4) a) Solve $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ for the given square mesh with boundary conditions as given. **[5]**



b) Find the numerically largest eigen values and corresponding eigen vectors for the following matrix $A=\begin{bmatrix}1 & 3 & -1\\ 3 & 2 & 4\\ -1 & 4 & 10\end{bmatrix}$ by power method. **[5]**

Q5) a) Given $\frac{\partial^2 f}{\partial x^2}-\frac{\partial f}{\partial t}=0;\ f(0,t)=f(5,t)=0,\ f(x,0)=x^2(25-x^2)$; Find the values of f for x and t with h = 1, using Bendre's Schmidt method. **[5]**

b) Using Rayleigh-Ritz Method solve the boundary value problem $y''-y+4xe^x=0, y(0)=0=y(1)$. **[5]**

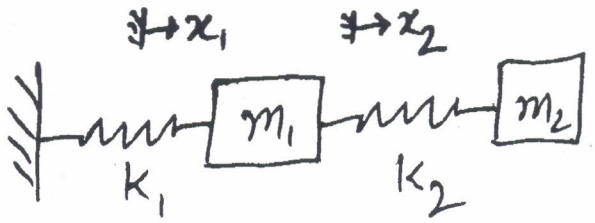
Q6) a) Find the extremal of $\int_1^2 \frac{x^3}{(y')^2}\,dx$ with $y(1)=0, y(2)=3$. **[5]**

b) Solve the system of equations by least square method $x - y = 2$, $x + y = 4$, $2x + y = 8$. **[5]**

Q7) a) Find the image of the triangular region bounded by $x = 0, y = 0, x + y = 1$ under the transformation $w = z^2$. **[5]**

b) Find the Fourier sine transformation of the function $f(x) = e^{-|x|}$. Hence evaluate $\int_0^\infty \frac{x \sin mx}{1+x^2}\,dx$. **[5]**

Q8) a) For the system of masses & spring in the figure below $m_1 = 2$, $m_2 = 1$, $k_1 = 4$ and $k_2 = 2$, assuming there is no friction. Find natural frequencies of the system and corresponding normal modes of vibration using matrix method **[5]**



b) The function U satisfies the equation $\frac{\partial^2 U}{\partial t^2} = \frac{\partial^2 U}{\partial x^2}$, $0 \le x \le 1$; subject to the initial conditions $u(x,0) = \sin \pi x$, $0 \le x \le 1$ & $U_t(x,0) = 0$ with boundary conditions $u(0,t) = u(1,t) = 0, t > 0$ by taking h = 0.2 upto five levels. **[5]**