Total No. of Questions—**12**] [Total No. of Printed Pages—**8+1**

Seat No.	

[4857]-201

S.E. (Computer/IT) (First Semester) EXAMINATION, 2015

DISCRETE STRUCTURES

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

N.B. **:—** (*i*) Attempt Q. No. **1** *or* Q. No. **2,** Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6** from Section I.

(*ii*) Attempt Q. No. **7** *or* Q. No. **8,** Q. No. **9** *or* Q. No. **10**, Q. No. **11** *or* Q. No. **12** from Section II.

(*iii*) Answers to two sections should be written in separate answer-books.

(*iv*) Neat diagrams must be drawn wherever necessary.

(*v*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Find CNF and DNF for the following formulae given below : [6]

(*i*) $(p \vee \overline{q}) \rightarrow q$

(*ii*) $p \leftrightarrow (\overline{p} \vee \overline{q})$.

(*b*) Prove that $5^n - 1$ is divisible by 4 for all $n \geq 1$. [6]

(*c*) Among a set of integers 1 to 500. Find how many of these nos. are divisible by 3 or by 5 or by 11. How many are divisible by 3 or 11 but not by 5 ? [6]

P.T.O.

Or

2. (*a*) In the class of 55 students, the no. of students studying different subjects are as given below :

Maths 23, Physics 24, Chemistry 19, Maths + Physics 12, Maths + Chemistry 9, Physics + Chemistry 7, all three subjects 4. Find the no. of students who have taken :

(*i*) at least one subject

(*ii*) exactly one subject

(*iii*) exactly two subjects. [6]

(*b*) Construct truth table to determine whether each of the following is tautology or contradiction : [6]

(*i*) $p \vee \sim (p \wedge q)$

(*ii*) $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$.

(*c*) Determine the validity of argument given : [6]

S_1 : If I like mathematics then I will study

S_2 : Either I will study or I will fail

S : If I fail then I do not like Mathematics.

3. (*a*) Define the following terms with examples : [6]

(*i*) Ring

(*ii*) Field

(*iii*) Integral domain.

(*b*) Show that $<z, +>$ is a group where

$$z = \{....., -2, -1, 0, 1, 2, ...\}$$

set of all integer and binary operation on z is addition. [4]

(*c*) Define cummutative Ring and unity element in a Ring. Let z be a ring of integers : [6]

(*i*) Is z commutative

(*ii*) Does I have a unity element ?

(*iii*) What are the units in z ?

Or

4. (*a*) Let G be a group, for a fixed element G, let :

$$Gx = \{a \in G \mid ax = xa\}.$$

Show that Gx is a subgroup of G for all $x \in G$. [4]

(*b*) Define each of the following : [6]

(*i*) Groups

(*ii*) Normal subgroups

(*iii*) Homomorphism of groups.

(*c*) Show that $<z, *>$ is a group, where z is a set of all integers that are divisible by 2 and * is a binary operation giving multiplication of 2 integers. [6]

5. (*a*) Draw the Hasse diagram of the following sets under partial ordering relation "divides" and indicate those are in chains {1, 3, 9, 18}. [4]

(*b*) Let the functions f, g and h are defined as follows. Find rules for functions : [6]

(*i*) f o f

(*ii*) f o g

(*iii*) g o f

$f : R \to R,\ f(x) = 4x - 3$

$g : R \to R,\ g(x) = x^2 + 1$

$$h : R \to R,\ h(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}.$$

(*c*) Consider the set :

$$A = \{1, 4, 7, 13\} \text{ and}$$

$$R = \{(1, 4), (4, 7), (7, 4), (1, 13)\}.$$

Find out transitive closure of R using Wanhall's algorithm. [6]

Or

6. (*a*) Solve the recurrence relation : [6]

$$a_{n+2} - a_{n+1} - 6a_n = 0$$

(*b*) Let :

$$A = \{1, 2, 3, 4\} \text{ and}$$

$$R = \{(4, 1), (4, 3), (3, 2), (3, 3), (2, 2), (4, 2)\}$$

(*i*) Find reflexive closure of R.

(*ii*) Find symmetric closure of R. [4]

(*c*) Let

$$A = \{a, b, c, d\}$$

$$R_1 = \{(a, a), (b, b), (c, c) (a, b)\}$$

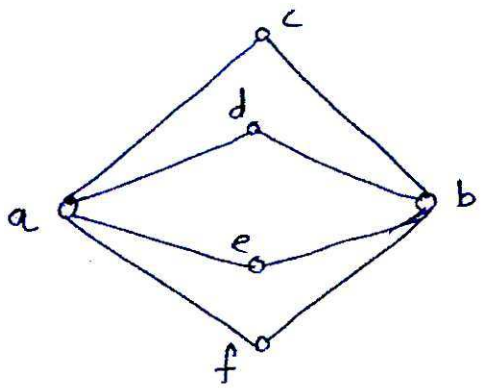
$$R_2 = \{(a, a), (b, d), (d, c)\}.$$

Find $(R_1 \cup R_2)$ and draw its diagraph. [6]

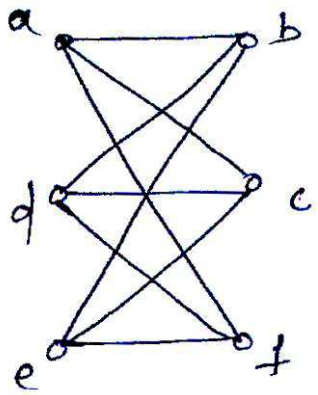
SECTION II

7. (*a*) Determine whether or not each of the graph is bipartite. Explain : [6]

(*i*)

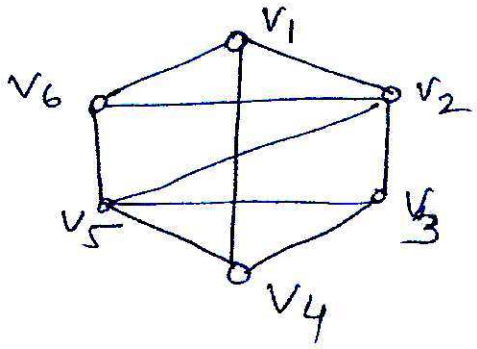


(*ii*)

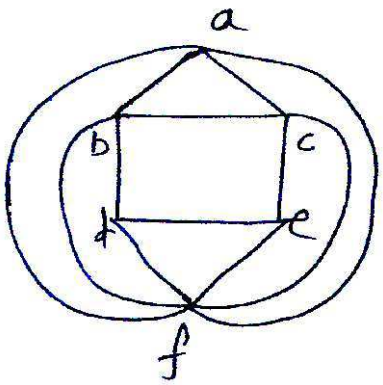


(*b*) Determine whether the following graphs are Hamiltonian or Eulerian path. [6]

(*i*)



(*ii*)

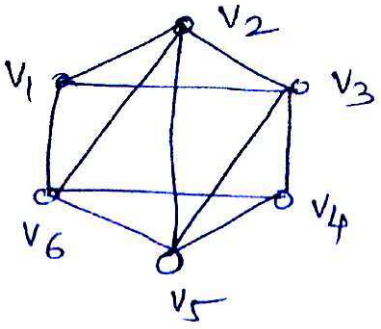


(*c*) Show that a simple planar graph has a vertex of degree 5 or less. [4]

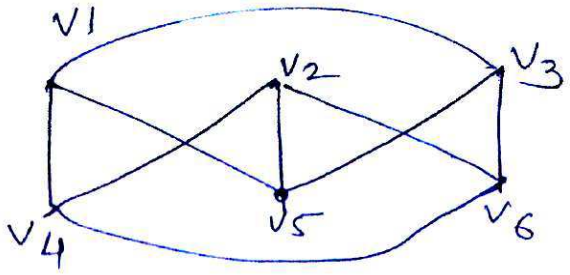
Or

8. (*a*) Draw planar representation of each graph. [6]

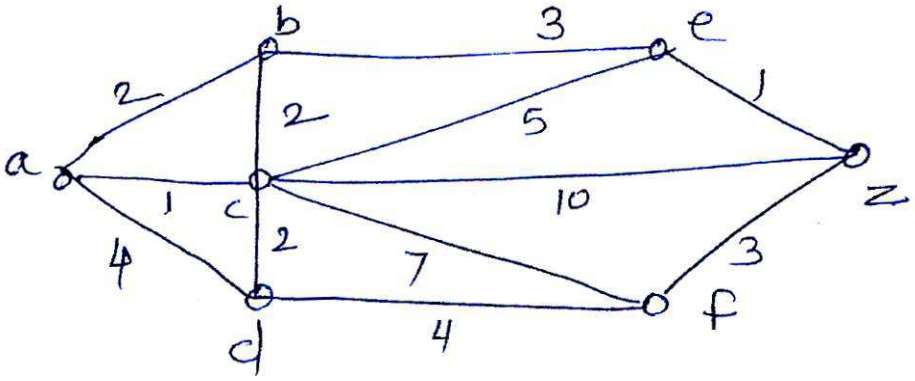
(*i*)



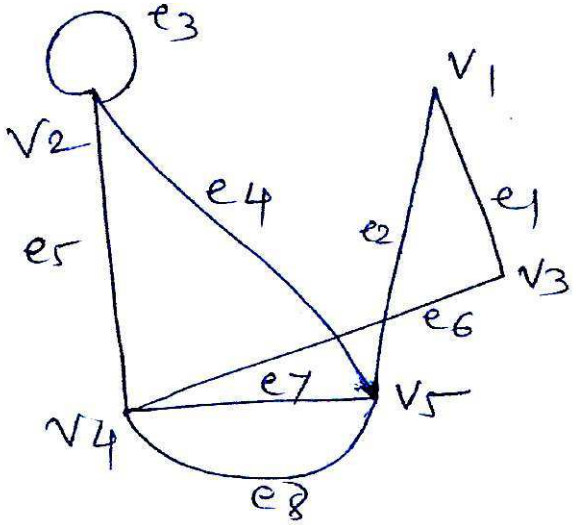
(*ii*)



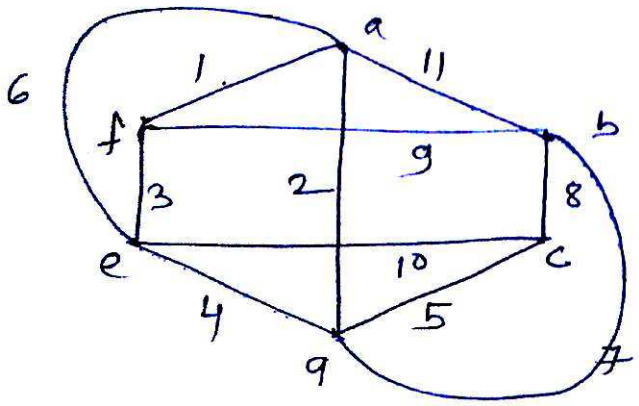
(*b*) Use Dijkstra's algorithm to find the shortest path from *a* to *z*. [6]



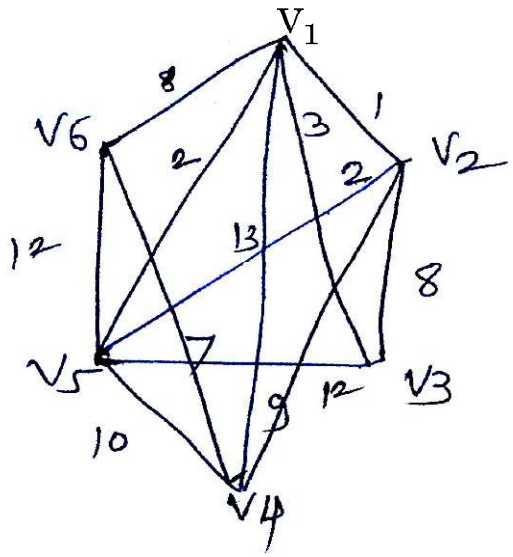
(*c*) Find the adjacency matrix, linked representation for the graph shown in the following figure. [4]



9. (*a*) Use Prim's algorithm to construct minimal spanning tree starting from vertex *a*. [6]



(*b*) Use Kruskal's algorithm for finding minimum spanning tree. For the given graph. [6]

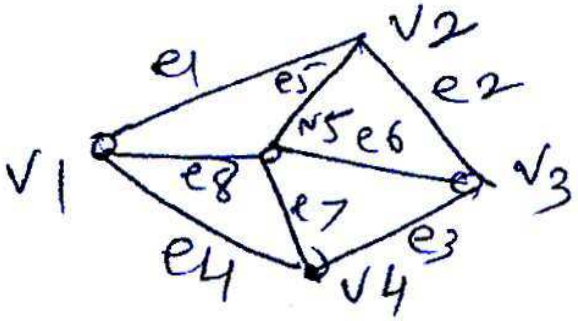


(*c*) Define spanning tree and minimum spanning tree with example. [4]

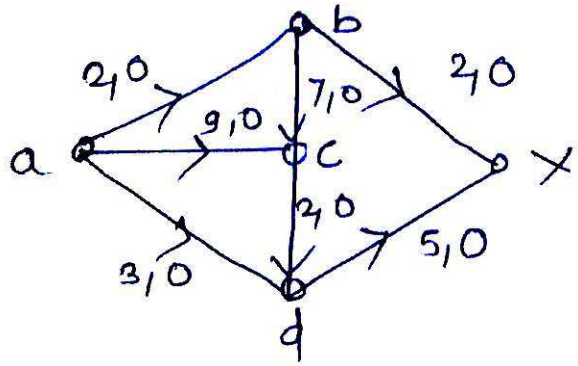
Or

10. (*a*) For the following set of weights, construct optimal binary prefix code. For each weight in the set, give corresponding code words 8, 9, 12, 14, 16, 19. [6]

(*b*) For the following figure, give fundamental circuits. [6]



(*c*) Find maximum flow in a transport n/w. [4]



11. (*a*) Out of 4 officers and 10 clerks, a committee of 2 officers, and 3 clerks is to be formed. In how many ways can committee be done if : [6]

(*i*) Any officer and any clerk can be included.

(*ii*) A particular clerk must be in committee.

(*iii*) A particular officer cannot be in committee.

(*b*) 12 persons are made to sit around a table. Find the number of ways they can sit such that 2 specific persons are not together. [6]

(*c*) Suppose repetitions are permitted : [6]

(*i*) How many ways three digit no. can be formed from six digits 2, 3, 4, 5, 7 and 9 ?

(*ii*) How many of these numbers are less than 400 ?

(*iii*) How many are even ?

(*iv*) How many are odd ?

(*v*) How many are multiple of 5 ?

(*vi*) How many are multiple of 10 ?

Or

12. (*a*) Suppose license plate contains 3 English letter followed by 4 digits : [6]

(*i*) How many different license plates can be manufactured if repetition of letters and digits are allowed ?

(*ii*) How many plates are possible if only the letters are repeated ?

(*iii*) How many plates are possible if the digits are repeated ?

(*b*) Find the no. of ways of arranging of the word TENNESSEE all at a time : [6]

(*i*) If there is a restriction

(*ii*) If the first two letters must be 'E'.

(*c*) In the experiment of rolling a die, find the probability of getting an odd number. [6]