Total No. of Questions—**12**] [Total No. of Printed Pages—**8**

Seat No.	

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S.E. (Electrical/Inst./Comp./I.T.) EXAMINATION, 2015

ENGINEERING MATHEMATICS—III

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

***N.B.* :—** (*i*) Answer Q. No. **1** *or* Q. No. **2,** Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6,** Q. No. **7** *or* Q. No. **8**, Q. No. **9** *or* Q. No. **10** Q. No. **11** *or* Q. No. **12**.

(*ii*) Answers to the two sections should be written in separate answer-books.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Figures to the right indicate full marks.

(*v*) Use of logarithmic tables, non-programmable electronic pocket calculator is allowed.

(*vi*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Solve any *three* of the following : [12]

(*i*) $(D^2 - 4D + 4)\, y = e^{2x} \sin 3x$

(*ii*) $[x^2D^2 - 2xD - 4]\, y = x^2 + 2 \log x$

(*iii*) $(D^2 - 2D)\, y = e^x \sin x$ (by method of variation of parameters.)

(*iv*) $(D^2 - 6D + 9)\, y = \dfrac{e^{3x}}{x^2}$.

(*b*) A capacitor of 10^{-3} farads is in series with an e.m.f. of 20 V and an inductor of 0.4 H. At time $t = 0$, the charge q and current i are zero. Find the charge q at any time t. [5]

Or

2. (*a*) Solve any *three* of the following : [12]

(*i*) $(D^2 - 4D + 3)\, y = x^3.e^{2x}$

(*ii*) $(D^2 + 5D + 6)\, y = e^{-2x} \sin 2x$

(*iii*) $[x^3D^3 + 2x^2D^2 + 2]\, y = 10\left(x + \frac{1}{x}\right)$

(*iv*) $\dfrac{dx}{x(2y^4 - z^4)} = \dfrac{dy}{y(z^4 - 2x^4)} = \dfrac{dz}{z(x^4 - y^4)}$.

(*b*) Solve the system of equation : [5]

$$\frac{dx}{dt} - wy = a \cos pt;$$

$$\frac{dy}{dt} + wx = a \sin pt \quad (w \neq p).$$

3. (*a*) If [6]

$$f(z) = u + iv$$

is analytic, find $f(z)$ if :

$$u - v = (x - y)(x^2 + 4xy + y^2).$$

(*b*) Evaluate : [5]

$$\int_c \frac{\sin \pi z^2 + 2z}{(z-1)^2 (z-2)}\, dz.$$

where '*c*' is circle $|z| = 4$.

(c) Show that under the transformation [5]

$$w = \frac{i - z}{i + z},$$

x-axis in z plane is mapped onto the circle

$$u^2 + v^2 = 1.$$

Or

4. (a) If [5]

$$u = \log (x^2 + y^2)^{1/2},$$

find v such that $f(z)$ is analytic and

$$f(z) = u + iv.$$

(b) Evaluate : [6]

$$\int_c \frac{e^z}{(z+1)^3 (z-1)^2} dz$$

where c is contour $|z + 1| = \frac{1}{2}$.

(c) Find the bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = i, 0, -i$. [5]

5. (a) Find Fourier cosine integral of the function : [6]

$$f(x) = e^{-x} \cos x, \ x > 0.$$

(b) Find Fourier sine transform of : [5]

$$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2-x, & 1 \le x \le 2 \\ 0, & x > 2. \end{cases}$$

(c) Find z transform of the following (any *two*) : [6]

(i) $f(k) = \left(\frac{1}{2}\right)^{|k|}$ for all k

(*ii*) $f(k) = e^{-ak} \cos bk, \ k \geq 0$

(*iii*) $f(k) = \frac{\sin ak}{k}, \ k > 0.$

Or

6. (*a*) Solve : [5]

$$f(k+2) + 3f(k+1) + 2f(k) = 0$$
$$f(0) = 0, \ f(1) = 1.$$

(*b*) Find Inverse z transform (any *two*) : [6]

(*i*) Use inversion integral method to find :

$$\left[\frac{10z}{(z-1)(z-2)}\right]$$

(*ii*) $\frac{z^2}{z^2+4}, \ |z| > 2$

(*iii*) $\frac{z}{z^2-5z+6}, \ 2 < |z| < 3$

(*c*) Solve the integral equation : [6]

$$\int_0^\infty f(x) \cos \lambda x \, dx = e^{-\lambda}, \ \lambda > 0.$$

SECTION II

7. (*a*) The first four moments about the assumed mean 30 of a distribution are 0.3, 6, 30 and 400. Calculate the moments about the mean, standard deviation, mean and coefficient of skewness and kurtosis. [9]

(*b*) The regression equations are [8]

$$4x - 5y + 33 = 0 \text{ and}$$
$$20x - 9y - 107 = 0.$$

If the variance of y is 16, then find :

(*i*) mean of x and y

(*ii*) correlation coefficient between x and y

(*iii*) variance of x.

Or

8. (*a*) Probability of man now aged 60 years will live upto 70 years is 0.75. Find the probability that out of 10 men at 60 years old at least 3 will live upto 70 years. [5]

(*b*) The average number of misprints per page of a book is 2.5. Assuming the distribution of number of misprints to be Poisson, find :

(*i*) the probability that a particular book is free from misprints

(*ii*) number of pages containing more than one misprints if the book contain 900 pages. [6]

(*c*) In a test on 2000 electric bulbs it was found that the life of a particular make was normally distributed with average time 2040 hours and standard deviation of 60 hours. Estimate the number of bulbs likely to burn for more than 1920 hours but less than 2160 hours.

(Given : $z = 2$, Area $= 0.4772$). [6]

9. (*a*) Prove that : [4]

$$\frac{d}{dt}\left[\bar{v}.\frac{d\bar{v}}{dt} \times \frac{d^2\bar{v}}{dt^2}\right] = \bar{v}.\frac{d\bar{v}}{dt} \times \frac{d^3\bar{v}}{dt^3}.$$

(*b*) Find the directional derivative of

$$\phi = x^2 - y^2 + z^2$$

at the point P(1, 2, 3) in the direction of $\overline{PQ}$, where Q is the point (5, 0, 4). [6]

(*c*) Prove the following (any *two*) : [6]

(*i*) $\nabla.\left(r\,\nabla\left(\frac{1}{r^n}\right)\right) = \frac{n(n-2)}{r^{n+1}}$

(*ii*) $\nabla^2\left(\nabla.\left(\frac{\overline{r}}{r^2}\right)\right) = \frac{2}{r^4}$

(*iii*) $\nabla \times (\overline{a} \times \overline{r}) = 2\overline{a}$, for a constant vector $\overline{a}$.

Or

10. (*a*) Find the angle between tangents to the curve [4]

$$\overline{r} = (t^3 + 2)i + (4t - 5)j + (2t^2 - 6t)k$$

at $t = 0$ and $t = 2$.

(*b*) If the directional derivative of : [6]

$$\phi = ax^2y + by^2z + cz^2x$$

at (1, 1, 1) has maximum magnitude 12 in the direction parallel to the line

$$\frac{x-1}{1} = \frac{y-3}{2} = \frac{z}{2},$$

then find the value of *a*, *b*, *c*.

(*c*) Show that : [6]

$$\overline{F} = (x + 2y + 4z)i + (2x - 3y - z)j + (4x - y + 2z)k$$

is irrotational. Find scalar potential ϕ such that $\overline{F} = \nabla\phi$.

11. (*a*) Evaluate the integral : [5]

$$\int_c \overline{F} \, . \, d\overline{r} \, ,$$

along the straight line joining the points (1, –2, 1) and (3, 1, 4), where

$$\overline{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k} \, .$$

(*b*) Evaluate [6]

$$\iint_S \left(\nabla \times \overline{F}\right) . \, \hat{n} \, dS \, ,$$

using Stokes' theorem' where

$$\overline{F} = (x^3 - y^3)\hat{i} - xyz\,\hat{j} + y^3\hat{k} \, ,$$

and 'S' is the surface

$$x^2 + 4y^2 + z^2 - 2x = 4,$$

above the plane $x = 0$.

(*c*) Evaluate [6]

$$\iint_S \overline{F} \bullet \hat{n} \, dS \, ,$$

where

$$\overline{F} = yz\,\hat{i} + xz\,\hat{j} + xy\,\hat{k} \, ,$$

and S is the surface of the sphere

$$x^2 + y^2 + z^2 = 1,$$

in the positive octant.

Or

12. (*a*) Apply Green's theorem to evaluate : [5]

$$\int_c \overline{F} \cdot d\overline{r},$$

where

$$\overline{F} = \cos y\,\hat{i} + x(1 - \sin y)\,\hat{j}$$

and '*c*' is the curve

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

in the plane $z = 0$.

(*b*) Evaluate [6]

$$\iint_S \left(\nabla \times \overline{F}\right) \cdot \hat{n}\, dS,$$

where

$$\overline{F} = x^2\,\hat{i} + xy\,\hat{j},$$

and S is the surface of a Triangular lamina bounded by $x = 1$, $y = 1$, $x + y = 3$.

(*c*) Evaluate

$$\iint_S \overline{r} \cdot \hat{n}\, dS,$$

over the surface of a sphere of radius '*a*' with the centre at origin. [6]