Total No. of Questions—**12**] [Total No. of Printed Pages—**8**

Seat No.	

[4857]-114

S.E. (Mech./Auto./Prod. S/W/ Indus) (First Semester)

EXAMINATION, 2015

ENGINEERING MATHEMATICS—III

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

***N.B.* :—** (*i*) Answer Q. No. **1** *or* Q. No. **2,** Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6** from Section I and Q. No. **7** *or* Q. No. **8,** Q. No. **9** *or* Q. No. **10**, Q. No. **11** *or* Q. No. **12** from Section II.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Figures to the right indicate full marks.

(*v*) Use of non-programmable electronic pocket calculator is allowed.

(*vi*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Solve any *three* : [12]

(*i*) $(D - 1)^3 y = e^x + 3$

(*ii*) $(D^3 + 4D)y = \sin 2x$

(*iii*) $(D^2 - 4D + 3)y = x^3 e^{2x}$

(*iv*) $(D^2 + 1)y = \text{cosec } x$ (by variation of parameters).

(*b*) Solve :

$$\frac{dx}{dt} + y = \sin t$$

$$\frac{dy}{dt} + 4x = \cos t$$

Given that $x = 0$, $y = 1$ at $t = 0$. [5]

Or

2. (*a*) Solve any *three* : [12]

(*i*) $(D - 1)^2 y = x^2 e^x$

(*ii*) $(D^2 - 1)y = x \sin 3x$

(*iii*) $x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{x}\sin(\log x)$

(*iv*) $\frac{dx}{y} = \frac{dy}{-x} = \frac{dz}{2x - 3y}$.

(*b*) A 3 N weight stretches a spring 15 cms. If the weight is pulled 10 cms below the equilibrium position and then released. Find the displacement as a function of time *t*. [5]

3. (*a*) Find Laplace Transform of the following (any *two*) : [6]

(*i*) $F(t) = \begin{cases} a, & 0 < t < b \\ 0, & t > b \end{cases}$

(*ii*) $\int_0^t \left(\frac{e^t - \cos 2t}{t}\right) dt$

(*iii*) $\frac{e^{at} - e^{bt}}{t}$.

(*b*) Solve by Laplace transform method : [5]

$$y'' + y = 0$$

given $y(0) = 1;\ y'(0) = 1$.

(*c*) By considering Fourier cosine transform of e^{-mx} $(m > 0)$ prove that : [6]

$$\int_0^{\infty} \frac{\cos \lambda x}{\lambda^2 + m^2}\, d\lambda = \frac{\pi}{2m} e^{-mx}; \quad m > 0;\ x > 0.$$

Or

4. (*a*) Find inverse Laplace transform (any *two*) : [8]

(*i*) $\dfrac{1}{s\left(s^2 + 4\right)}$

(*ii*) $\dfrac{s}{s^2 + 6s + 25}$

(*iii*) $\dfrac{1}{(s+1)\left(s^2 + 1\right)}$

by convolution theorem.

(*b*) Find Fourier sine transform of $\dfrac{e^{-ax}}{x}$. [4]

(*c*) Using Fourier integral representation show that : [5]

$$\int_0^{\infty} \frac{2\cos \lambda x}{1 + \lambda^2}\, d\lambda = \begin{cases} 0 & ; x < 0 \\ \pi e^{-x} & ; x > 0 \end{cases}$$

5. (*a*) Find the deflection $y(x, t)$ of the vibrating string of length π and ends are fixed, corresponding to zero initial velocity.

Given deflection : [8]

$$f(x) = y = k(\sin x - \sin 2x)$$

at $t = 0$; use $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$.

(*b*) Solve :

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2},$$

for the conduction of heat along a homogeneous rod of length l with insulated sides is initially at a uniform temperature u_0. Its ends are cooled to 0°C and are kept at that temperature. Find the temperature $u(x, t)$ at any time t. [8]

Or

6. (*a*) Solve the equation : [8]

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

with condition :

(*i*) $u = 0$ when $y \to \infty$; $\forall x$

(*ii*) $u = 0$ when $x = 0$, $\forall y$

(*iii*) $u = 0$ when $x = 1$, $\forall y$

(*iv*) $u = (x - x^2)$, when $y = 0$; for $0 \le x \le 1$.

(*b*) Use Fourier transform to solve : [8]

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}; \quad 0 < x < \infty; \ t > 0$$

under the conditions :

(*i*) $u(0, t) = 0$; $t > 0$

(*ii*) $u(x, 0) = \begin{cases} 1; & 0 < x < 1 \\ 0; & x > 1 \end{cases}$

(*iii*) $u(x, t)$ is bounded.

SECTION II

7. (*a*) The first four moments about any point are 0, 2.5, 0.7 and 18.75. Calculate the first four moment about mean. Also find β_1 and β_2. [6]

(*b*) Given :

$$n = 10, \Sigma x_i = 299, \Sigma y_i = 290, \Sigma x_i y_i = 9403,$$

$$\Sigma x_i^2 = 9849, \Sigma y_i^2 = 9104.$$

Calculate the co-efficient of correlation. [5]

(*c*) From a deck of 52 cards, two cards are drawn at random. Find the probability that : [5]

(*i*) both are hearts

(*ii*) both the cards are of different series.

Or

8. (*a*) For random variables, if P(1) = P(2) then by Poisson distribution find mean and the value of P(4). [5]

(*b*) In a sample of 1000 cases, the mean of certain test is 14 and S.D. is 25. Assuming the distribution is normal then how many students score below 8 marks.

Given $A(z = 0.24) = 0.0948$. [5]

(*c*) The equation of two regression lines are :

$$2x + 3y = 8 \text{ and}$$
$$x + 2y = 5$$

Find the mean values of x and y. Obtain the value of correlation co-efficient. [6]

9. (*a*) If [4]

$$\bar{r} = \bar{a} \sinh t + \bar{b} \cosh t,$$

where $\bar{a}$ and $\bar{b}$ are constant vectors, then find the value of :

$$\frac{d\bar{r}}{dt} \times \frac{d^2\bar{r}}{dt^2}$$

(*b*) Find the directional derivative of : [5]

$$\phi = xy^2 + yz^3$$

at point (2, –1, 1) in the direction of vector :

$$\bar{i} + 2\bar{j} + 2\bar{k}$$

(*c*) Prove that : [8]

(*i*) $\nabla^4 r^4 = 120$

(*ii*) If ϕ and ψ satisfy Laplace equation, then prove that :

$$(\phi \nabla \psi - \psi \nabla \phi)$$

is solenoidal vector.

Or

10. (*a*) Show that the vector field : [6]

$$\bar{F} = \left(2xz^3 + 6y\right)\bar{i} + (6x - 2yz)\bar{j} + \left(3x^2z^2 - y^2\right)\bar{k}$$

is irrotational vector field. Also find the corresponding scalar potential ϕ such that $\bar{F} = \nabla\phi$.

(*b*) Find the directional derivative of :

$$\phi = 2xz^4 - x^2y$$

at (2, –2, 1) towards point (1, 1, –1). [6]

(*c*) Find the angle between the tangents to the curve : [5]

$$x = t^2 + 1$$

$$y = 4t - 3$$

$$z = 2t^2 - 6t$$

at $t = 2$ and $t = 3$.

11. (*a*) Evaluate : [5]

$$\int_C \overline{F} \cdot d\overline{r}; \text{ for}$$

$$\overline{F} = 3x^2\overline{i} + (2xz - y)\overline{j} + z\overline{k}$$

along the straight line joining (0, 0, 0) to (1, 2, 3).

(*b*) Evaluate : [6]

$$\iint (\nabla \times \overline{F}) \cdot d\overline{s}$$

by Stokes' theorem for the force field

$$\overline{F} = (x - y^2)\overline{i} + 2xy\overline{j}$$

in the plane $z = 0$, bounded by $y = 0$, $x = 2$, $y = x$.

(*c*) By Divergence theorem, evaluate : [6]

$$\iint_S \left(\frac{x\overline{i} + y\overline{j} + z\overline{k}}{r^2} \right) \cdot d\overline{s}$$

where s is the surface bounded by sphere

$$x^2 + y^2 + z^2 = a^2.$$

Or

12. (*a*) Evaluate : [6]

$$\int_C \left[\left(2x^2 - y\right)\bar{i} + \left(\tan y - e^y + 4x\right)\bar{j}\right] . d\bar{r}$$

where, C : square with sides of length 5. Use Green's theorem.

(*b*) Evaluate : [6]

$$\int_C \left(xy\,dx + xy^2\right)dy\ ;$$

by Stokes' theorem, where, C : the square in *xy* plane with vertices (0, 0), (1, 0), (0, 1).

(*c*) Velocity distribution for a fluid is given by : [5]

$$u = -x;\ v = 2xy;\ w = 3 - z$$

Determine the equations of stream lines passing through the point (1, 1, 0).