Total No. of Questions—**8**] [Total No. of Printed Pages—**4+1**

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# [4857]-1005

## S.E. (Civil) (First Semester) EXAMINATION, 2015

## ENGINEERING MATHEMATICS-III

## (2012 PATTERN)

**Time : Two Hours** **Maximum Marks : 50**

***N.B.*** **:—** (*i*) Answer Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4,** Q. No. **5** *or* Q. No. **6** and Q. No. **7** *or* Q. No. **8**

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of non-programmable electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

**1.** (*a*) Solve any *two* : [8]

(*i*) $(D^2 - 9D + 18)y = e^{e^{-3x}}$

(*ii*) $(D^3 + 3D^2 - 4)y = x^2 + x + 1$

(*iii*) $x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = \sin(2 \log x)$.

(*b*) Solve the following system of linear equations by Gauss-Seidel method : [4]

$$28x_1 + 4x_2 - x_3 = 32$$

$$2x_1 + 17x_2 + 4x_3 = 35$$

$$x_1 + 3x_2 + 10x_3 = 24$$

P.T.O.

*Or*

**2.** (*a*) Solve the following system of symmetrical simultaneous equation : [4]

$$\frac{dx}{1} = \frac{dy}{3} = \frac{dz}{5z + \tan(y - 3x)}.$$

(*b*) Use Runge-Kutta method of fourth order to obtain the numerical solution of :

$$\frac{dy}{dx} = x^2 + y^2, \; y(1) = 1.5$$

in the interval (1, 1.2) with $h$ = 0.2. [4]

(*c*) Solve the following system by Cholesky method : [4]

$$4x_1 - 2x_2 = 0$$

$$-2x_1 + 4x_2 - x_3 = 1$$

$$-x_2 + 4x_3 = 0$$

**3.** (*a*) The first four moments of a distribution about the value 5 are 3, 30, 50 and 60. Obtain the first four central moments and coefficient of skewness and kurtosis. [4]

(*b*) Given a normal distribution with $\mu$ = 50 and $\sigma$ = 10, find the probability that $x$ assumes a value between 45 and 62. (Given that : P($z$ > 0.5) = 0.3085, P($z$ < 1.2) = 0.8849] [4]

(*c*) Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at (2, –1, 2) in the direction $2\bar{i} - 3\bar{j} + 6\bar{k}$. [4]

*Or*

**4.** (*a*) Attempt any *one* : [4]

(*i*) Show that :

$$\nabla^2 \left[ \nabla \cdot \left( \frac{\bar{r}}{r} \right) \right] = 0$$

(*ii*) If $\bar{u}$ and $\bar{v}$ are irrotational vectors then prove that $\bar{u} \times \bar{v}$ is solenoidal vector.

(*b*) Verify whether the following field is irrotational : [4]

$$\bar{F} = (6xy + z^3)\, \bar{i} + (3x^2 - z)\, \bar{j} + (3xz^2 - y)\, \bar{k}.$$

(*c*) Calculate the coefficient of correlation from the following data : [4]

$$n = 20,\ \Sigma x = 40,\ \Sigma x^2 = 190,\ \Sigma y^2 = 200,\ \Sigma xy = 150,\ \Sigma y = 40.$$

**5.** (*a*) Prove that :

$$\bar{F} = (4xy - 3x^2 z^2)\, \hat{i} + 2x^2 \hat{j} - 2x^3 z\, \hat{k}$$

is a conservative field and also find the work done in moving an object in this field from (0, 0, 0) to (1, 1, 1). [5]

(*b*) Use divergence theorem to evaluate :

$$\iint_S \bar{F} \cdot d\bar{S}, \quad \text{where} \quad \bar{F} = x^3 \hat{i} + x^2 y \hat{j} + x^2 z \hat{k}$$

and S is the surface bounding the region $x^2 + y^2 = a^2$, $z = 0$ and $z = b$. [4]

(*c*) Evaluate : [4]

$$\iint_S \nabla \times \bar{F} \cdot d\bar{S} \quad \text{for} \quad \bar{F} = y\hat{i} + z\hat{j} + x\hat{k}$$

where S is the surface of the paraboloid $z = 1 - x^2 - y^2,\ z \geq 0$.

*Or*

**6.** (*a*) Use Green's Lemma to evaluate the line integral : [5]

$$\oint_C (\cos x \sin y - 4y)dx + \sin x \cos y \, dy,$$

where C is the circle $x^2 + y^2 = 1$.

(*b*) Evaluate :

$$\iint_S 3x \; dy\,dz - 2y\,dz\,dx + 2z \; dx\,dy)$$

over the surface of a sphere of radius *a*. [4]

(*c*) Evaluate : [4]

$$\iint_S \nabla \times \overline{F} \, . \, \hat{n} \;\; ds$$

for the surface of the paraboloid :

$z = 4 - x^2 - y^2 \; (z \geq 0)$ and $\overline{F} = y^2 \hat{i} + z\hat{j} + xy\hat{k}$.

**7.** (*a*) Solve the equation : [7]

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

with conditions :

(*i*) $u = 0$ when $y = \infty$

(*ii*) $u = 0$ when $x = 0$

(*iii*) $u = 0$ when $x = 1$

(*iv*) $u = x(1 - x)$ when $y = 0$ for $0 \leq x \leq 1$

(*b*) Solve $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ if : [6]

(*i*) $u(0, t) = 0$

(*ii*) $u(l, t) = 0$

(*iii*) $u(x, 0) = u_0, \; 0 \leq x \leq l$

*Or*

**8.** (*a*) A thin sheet of metal, bounded by $x$-axis and the lines $x = 0$ and $x = 1$ and streching to infinity in the $y$ direction has its upper and lower faces perfectly insulated and its vertical edges and the edge at infinity are maintained at 0°C, while over the base temperature of 100°C. Find steady state temperature $u(x, y)$. [6]

(*b*) A string is stretched and fastened to two points $l$ aparts. Motion is started by displacing the string in the form $u = a \sin \frac{\pi x}{l}$ from which it is released at time $t = 0$. Find the displacement $u(x, t)$ from one end. [7]