Seat No.	

[4857]-1015

S.E. (Mechanical/Automobile/Mechanical Sandwich/Prod/Prod. Sand. Industrial) (First Semester) EXAMINATION, 2015

ENGINEERING MATHEMATICS—III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Neat diagrams must be drawn wherever necessary.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.

(*iv*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* of the following : [8]

(*i*) $(D^2 + 4D + 4)y = \dfrac{e^{-2x}}{x^2}$.

(*ii*) $(D^2 + 1)\, y = \operatorname{cosec} x$ (using method of variation of parameters)

(*iii*) $(x^2D^2 - xD + 1)y = x \log x$.

(*b*) Using Fourier Integral representation, show that : [4]

$$\int_0^\infty \frac{\cos\dfrac{\pi\lambda}{2} . \cos \lambda x}{1-\lambda^2} . \, d\lambda = \begin{cases} \dfrac{\pi}{2}\cos x, & |x| \le \dfrac{\pi}{2} \\ 0, & |x| > \dfrac{\pi}{2} \end{cases}$$

Or

2. (*a*) The weight of '1N', stretches a spring by 5 cm. A weight of '3N' is attached to the spring and weight 'W' is pulled '10 cm' below the equilibrium position and released. Determine the position and velocity as function of time. [4]

(*b*) Solve any *one* of the following : [4]

(*i*) Find the Laplace transform of $f(t) = \left\{\frac{e^{-at} - e^{-bt}}{t}\right\}$.

(*ii*) Find the inverse Laplace transform of :

$$F(s) = \left\{\frac{3s + 1}{(s - 1)(s^2 + 1)}\right\}.$$

(*c*) Solve the following differential equation by using Laplace transform method : [4]

$$y''(t) - 3y'(t) + 2y(t) = 12.e^{-2t}$$

Given that :

$$y(0) = 2, \ y'(0) = 6.$$

3. (*a*) Find the lines of regression for the following data : [4]

x	*y*
2	2
3	5
5	8
7	10
9	12
10	14
12	15
15	16

(*b*) A random sample of 200 screws is drawn from a population which represent size of screws. If a sample is distributed normally with a mean 3.15 cm and standard deviation 0.025 cm, find the expected number of screws whose size fall between 3.12 cm and 3.2 cm.

Area corresponding to 1.2 $\rightarrow$ 0.3849

Area corresponding to 2.0 $\rightarrow$ 0.4772 [4]

(*c*) Find the directional derivative of the function $\phi = e^{2x-y-z}$ at (1, 1, 1) in the direction of normal to the surface $x^2 + y^2 + z^2 = 9$ at (1, 2, 2). [4]

Or

4. (*a*) Calculate the correlation coefficient for the following data : [4]

x	***y***
6	9
2	11
10	5
4	8
8	7

(*b*) Attempt any *one* : [4]

(*i*) For a solenoidal vector field $\bar{\varepsilon}$, show that curl curl curl curl $\bar{\varepsilon} = \nabla^4 \bar{\varepsilon}$.

(*ii*) Prove that :

$$\bar{b} \times \nabla\left[\bar{a} \cdot \nabla \log r\right] = \frac{\bar{b} \times \bar{a}}{r^2} - \frac{2(\bar{a} \cdot \bar{r})}{r^4}(\bar{b} \times \bar{r}).$$

(*c*) Show that the vector field given by : [4]

$$\bar{F} = (y^2 \cos x + z^2)i + (2y \sin x)j + 2xzk$$

is irrotational. Find the scalar field ϕ such that $\bar{F} = \nabla\phi$.

5. (*a*) Evaluate $\int_C \bar{F} \cdot d\bar{r}$ for $\bar{F} = (2x + y)\bar{i} + (3y - x)\bar{j}$ and '*c*' is the straight line joining the points (0, 0) and (3, 2). [4]

(*b*) Evaluate $\iint_S \bar{F} \cdot d\bar{s}$, where $\bar{F} = yz\bar{i} + zx\bar{j} + xy\bar{k}$ and *s* is the part of the surface of the sphere $x^2 + y^2 + z^2 = 1$ which lies in the first octant. [4]

(*c*) Evaluate $\iint_S \text{curl}\, \bar{F} \cdot \hat{n}\, ds$ for the surface of the hemisphere $x^2 + y^2 + z^2 = a^2$ above the *xy* plane, where : [5]

$$\bar{F} = (x^2 + y - 4)\bar{i} + 3xy\bar{j} + (2xz + z^2)\bar{k}.$$

Or

6. (*a*) Verify Green's theorem for the field $\bar{F} = x^2\bar{i} + xy\bar{j}$ over the region R enclosed by $y = x^2$ and the line $y = x$. [5]

(*b*) Show that : [4]

$$\iiint_V \frac{dv}{r^2} = \iint_S \frac{\bar{r} \cdot \hat{n}}{r^2}\, dS.$$

(*c*) Evaluate $\iint_S (\nabla \times \bar{F}) \cdot \hat{n}\, dS$ for $\bar{F} = x^2\bar{i} + y^2 z\bar{j} + xy\bar{k}$ for the plane surface S bounded by $x = 0$, $y = 0$, $x = 2$, $y = 2$, $z = 0$. [4]

7. (*a*) A tightly stretched string with fixed end points $x = 0$ and $x = l$, is initially in a position given by $y(x, 0) = y_0 \sin^3 \frac{\pi x}{l}$. If it is released from rest from this position, find the displacement y at any distance x from one end at any time t with the differential equation : [7]

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}.$$

(*b*) Solve the equation $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$ where $u(x, t)$ satisfies the following conditions : [6]

(*i*) $u(0, t) = 0$

(*ii*) $u(l, t) = 0$ for all t

(*iii*) $u(x, 0) = x$ in $0 < x < l$.

Or

8. (*a*) An infinitely long plane uniform plate is bounded by two parallel edges in the y-direction and an end at right angles to them. The breadth of the plate is π. This end is maintained at temperature 40 at all points and other edges at zero temperature. Find the steady-state temperature function $u(x, y)$ [7]

(*b*) Using Fourier sine transform solve the partial differential equation : [6]

$$\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}, \ 0 < x < \infty, \ t > 0$$

subject to the following conditions :

(*i*) $u(0, t) = 0, \ t > 0$

(*ii*) $u(x, 0) = e^{-x}, \ x > 0$

(*iii*) u and $\frac{\partial u}{\partial x} \to 0$ as $x \to \infty$.