Total No. of Questions—**12**] [Total No. of Printed Pages—**8**

Seat No.	

[4756]-21

F.E. (Second Semester) EXAMINATION, 2015

ENGINEERING MATHEMATICS-II

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

N.B. **:—** (*i*) **Section I** : Solve Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**.

Section II : Solve Q. No. **7** *or* Q. No. **8**, Q. No. **9** *or* Q. No. **10**, Q. No. **11** *or* Q. No. **12**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of non-programmable electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Form the differential equation whose general solution is :

$$y = e^x [c_1 \cos x + c_2 \sin x],$$

where c_1 and c_2 are arbitrary constants. [6]

(*b*) Solve any *two* : [10]

(*i*) $\dfrac{dy}{dx} = (4x + y)^2$

(*ii*) $(x^2 - 3xy + 2y^2)\, dx + (3x^2 - 2xy)\, dy = 0$

(*iii*) $\dfrac{dy}{dx} + \dfrac{4x}{1 + x^2}\, y = \dfrac{1}{(x^2 + 1)^3}$

Or

2. (*a*) Form the differential equation by eliminating arbitrary constants c_1 and c_2 from the general solution given by : [6]

$$y = c_1 \cos \log x + c_2 \sin \log x$$

(*b*) Solve any *two* : [10]

(*i*) $\dfrac{dy}{dx} = \dfrac{6x - 4y + 3}{3x - 2y + 1}$

(*ii*) $\dfrac{dy}{dx} - \dfrac{\tan y}{1 + x} = (1 + x)\, e^x \sec y$

(*iii*) $(2x + e^x \log y)\, y\, dx + e^x\, dy = 0.$

3. Solve any *three* : [18]

(*a*) Find the orthogonal trajectories of the family :

$$xy = c^2$$

(*b*) An e.m.f. 200 e^{-5t} is applied to a series circuit consisting of 20 Ω resistor and 0.01 F capacitor. Find the charge and current at any time, assuming that there is no initial voltage on capacitor.

(*c*) A body of mass m falling from a rest is subject to the force of gravity and an air resistance proportional to the square of velocity (kv^2). If it falls through a distance x and possesses a velocity v at that instant. Prove that :

$$\frac{2kx}{m} = \log\left(\frac{a^2}{a^2 - v^2}\right)$$

where $mg = ka^2$.

(*d*) If 30% of radioactive substance disappeared in 10 days, how long will it take for 90% of it to disappeared ?

Or

4. Solve any *three* : [18]

(*a*) A body of temperature 100°C is placed in a room whose temperature is 20°C and cools to 60°C in 5 minutes. What is time required to reach temperature of body at 40°C.

(*b*) A voltage 10 e^{-2t} is applied at $t = 0$ to a circuit containing an inductance L and resistance R connected in a series. Find current I at any time t as a function of time t, given that when $t = 0$, $I = 0$.

(*c*) A long hollow pipe has an inner diameter of 10 cm and outer diameter of 20 cm, the inner surface is kept at 200°C and outer surface is at 50°C. The thermal conductivity $k = 0.12$. How much heat is lost per minute from the portion of the pipe 20 m long.

(*d*) A metal ball is heated to a temperature of 100°C at time $t = 0$, it is placed in a water which is maintained at 40°C. If the temperature of the ball reduces to 60°C in 4 minutes, find the time at which the temperature of ball is 50°C

5. (*a*) Find the Fourier series for :

$$f(x) = x^2, \quad -\pi < x < \pi$$

and hence deduce that : [9]

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \ldots\ldots = \frac{\pi^2}{12}.$$

(*b*) If :

$$U_n = \int_0^{\pi/4} \tan^n \theta \; d\theta$$

show that :

$$n\,(U_{n+1} + U_{n-1}) = 1$$

hence find U_4. [7]

Or

6. (*a*) The following table gives variation of periodic current over a period : [4]

t (sec)	**:**	0	T/6	T/3	T/2	2T/3	$\frac{5T}{6}$	T
A (amp)	:	1.98	1.30	1.05	1.30	–0.88	–0.25	1.98

Show that there is a direct current part of 0.75 amp in variable current and and obtain the amplitude of first harmonic. [8]

(*b*) Evaluate : [4]

$$\int_0^{\infty} e^{-2x^2} x^9 \, dx$$

(*c*) Prove that : [4]

$$\beta(m, n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} \, dx.$$

SECTION II

7. (*a*) Trace the following curves (any *two*) : [8]

(*i*) $xy^2 = a^2 (2a - x)$

(*ii*) $r = a(1 - \sin \theta)$

(*iii*) $x^{2/3} + y^{2/3} = a^{2/3}$

(*b*) Show that :

$$\phi(a) = \int_{\pi/6a}^{\pi/2a} \frac{\sin ax}{x} \, dx$$

is independent of a. [4]

(*c*) Find the perimeter of the cardioid : [5]

$$r = a (1 - \cos \theta)$$

Or

8. (*a*) Trace the following curves (any *two*) : [8]

(*i*) $y^2 (3a - x) = x^3$

(*ii*) $r = a \sin 2\theta$

(*iii*) $x = t^2, \; y = t - \frac{t^3}{3}$

(*b*) Show that : [4]

$$\int_0^\infty e^{-x^2 - 2ax} \, dx = \frac{\sqrt{\pi}}{2} e^{a^2} [1 - erf(a)]$$

(*c*) Find the whole length of the loop of the curve : [5]

$$3y^2 = x(x - 1)^2$$

9. (*a*) Prove that the sphere :

$$x^2 + y^2 + z^2 + 2x - 4y - 2z - 3 = 0$$

touches the plane :

$$2x - 2y - z + 16 = 0$$

and find the point of contact.

(*b*) Find the equation of the right circular cone whose vertex is at the origin, axis is the line :

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$$

and semi-vertical angle is 30°. [5]

(*c*) Find the equation of the right circular cylinder with radius 2 and axis is the line : [6]

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$$

Or

10. (*a*) Find the equation of the sphere which has its centre at (2, 3, –1) and which touches the line : [6]

$$\frac{x+1}{-5} = \frac{y-8}{3} = \frac{z-4}{4}$$

(*b*) Find the equation of the right circular cone with vertex (1, 2, 3) axis has direction ratios 2, –1, 4 and semi-vertical angle is 60°. [5]

(*c*) Find the equation of the right circular cylinder with axis :

$$\frac{x-1}{2} = \frac{y-3}{2} = \frac{z-5}{-1}$$

and radius = 3. [6]

11. Solve any *two* :

(*a*) Evaluate : [8]

$$\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dy\, dx}{(1+x^2+y^2)}$$

(*b*) Find the area of the upper half of the cardioid : [8]

$$r = a\,(1 + \cos\theta)$$

(*c*) Find the C.G of an arc of the catenary : [8]

$$y = a\cosh\left(\frac{x}{a}\right) \text{ from } x = -a \text{ to } x = a.$$

Or

12. Solve any *two* :

(*a*) Evaluate : [8]

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{y\,dy\, dx}{(1+y^2)\sqrt{1-x^2-y^2}}$$

(*b*) Evaluate : [8]

$$\int_0^{\log 2} \int_0^{x} \int_0^{x+y} e^{x+y+z}\, dx\, dy\, dz$$

(*c*) Find the moment of inertia of a sphere about a diameter. [8]