Total No. of Questions—8] [Total No. of Printed Pages—**4+2**

Seat No.	

[4756]-201

F.E. (Second Semester) EXAMINATION, 2015

ENGINEERING MATHEMATICS-II

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

N.B. **:—** (*i*) Attempt *four* questions : Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn whenever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of electronic non-programmable calculator is allowed.

(*v*) Assume suitable data whenever necessary.

1. (*a*) Solve the following differential equations : [8]

(*i*) $\frac{dy}{dx} = \cos x \cos y + \sin x \sin y$

(*ii*) $(x^2 + y^2 + 1)dx - 2xy\ dy = 0 \cdot$

(*b*) In a circuit containing inductance L, resistance R and voltage E, the current I is given by :

$$E = RI + L\frac{dI}{dt} \cdot$$

P.T.O.

Given :

L = 640 H, R = 250 Ω, E = 500 Volts.

I being zero when $t = 0$. Find the time that elapses before it reaches 80% of its maximum value. [4]

Or

2. (*a*) Solve : [4]

$$x \frac{dy}{dx} + y = y^2 \log x$$

(*b*) Solve the following : [8]

(*i*) A body at temperature 100°C is placed in a room whose temperature is 20°C and cools to 60°C in 5 minutes. Find its temperature after a further inrerval of 3 minutes.

(*ii*) A steam pipe 20 cm in diameter is protected with a covering 6 cm thick for which the coefficient of thermal conductivity is $k = 0.003$ cal/cm deg. sec in steady state. Find the heat lost per hour through a meter length of the pipe, if the surface of pipe is at 200°C and outer surface of the covering is at 30°C.

3. (*a*) Find a half range cosine series of $f(x) = \pi x - x^2$ in the interval $0 < x < \pi$. [5]

(*b*) Evaluate : [3]

$$\int_0^{\infty} \frac{x^3}{3^x} dx .$$

(*c*) Trace the following curve (any *one*) : [4]

(*i*) $y^2 = x^5 (2a - x)$

(*ii*) $r = a \sin 2\theta$.

Or

4. (*a*) If [4]

$$I_n = \int_{\pi/4}^{\pi/2} \cot^n \theta \, d\theta$$

prove that $I_n = \frac{1}{n-1} - I_{n-2}$. Hence evaluate I_3.

(*b*) Using differentiation under Integral sign prove that : [4]

$$\int_0^{\infty} \frac{e^{-x} - e^{-ax}}{x \sec x} dx = \frac{1}{2} \log \left(\frac{a^2 + 1}{2} \right)$$

for $a > 0$

(*c*) Find the length of the curve [4]

$$x = a(\theta - \sin\theta), \; y = a(1 - \cos\theta)$$

between $\theta = 0$ to $\theta = 2\pi$.

5. (*a*) Show that the plane $4x - 3y + 6z - 35 = 0$ is tangential to the sphere $x^2 + y^2 + z^2 - y - 2z - 14 = 0$ and find the point of contact. [5]

(*b*) Find the equation of the right circular cone whose vertex is given by (1, –1, 2) and axis is the line $\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-2}{-2}$ and semi-vertical angle is 45°. [4]

(*c*) Find the equation of right circular cylinder of radius 2 and axis is given by : [4]

$$\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z-3}{6}$$

Or

6. (*a*) Find the equation at the sphere through the circle $x^2 + y^2 + z^2 = 1,\ 2x + 3y + 4z = 5$ and which intersects the sphere $x^2 + y^2 + z^2 + 3(x - y + z) - 56 = 0$ orthogonally. [5]

(*b*) Find the equation of right circular cone with vertex at origin making equal angles with the co-ordinate axes and having generator with direction cosines proportional to 1, –2, 2. [4]

(*c*) Obtain the equation of the right circular cylinder of radius 5 where axis is : [4]

$$\frac{x-2}{3} = \frac{y-3}{1} = \frac{z+1}{1}.$$

7. Attempt any *two* of the following :

(*a*) Change the order of integration in the double integral : [6]

$$\int_0^5 \int_{2-x}^{2+x} f(x, y)\, dy\, dx$$

(*b*) Evaluate : [7]

$$\int_0^2 \int_0^x \int_0^{2x+2y} e^{x+y+z}\, dx\, dy\, dz$$

(*c*) Find the centroid of the loop of the curve : [6]

$$r^2 = a^2 \cos 2\theta .$$

Or

8. Attempt any *two* of the following :

(*a*) Evaluate : [6]

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} e^{-x^2-y^2}\, dx dy .$$

(*b*) Evaluate : [6]

$$\iiint \sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2}}\ dx\ dy\ dz$$

throughout the volume of ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

(*c*) Prove that the moment of inertia of the area included between the curves $y^2 = 4\,ax$ and $x^2 = 4ay$ about *x*-axis is $\frac{144}{35}\mathrm{M}a^2$, where M is the mass of the area included between the curves. [7]