Total No. of Questions : 8] SEAT No. :

P4129

[Total No. of Pages : 3

[4760] - 1064
M.E. (Mechanical) (Design Engineering)
ADVANCED MATHEMATICS
(Design / CAD-CAM / Auto Mobile)
(2013 Pattern)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Answer any five questions.*
2) *Neat diagrams must be drawn wherever necessary.*
3) *Figures to the right indicate full marks.*
4) *Use of non-programmable electronic pocket calculator is allowed.*
5) *Assume suitable data, if necessary.*

Q1) a) Find an orthonormal basis for the Euclidean space R^3, by applying Gram - schmidt's method to the following vectors : (1,0,1), (1,0,–1) and (0,3,4). **[5]**

b) If the potential function φ is $\log (x^2 + y^2)$, find the flux function ψ and the complex function $f(z) = \varphi + i\chi$. **[5]**

Q2) a) Evaluate $\oint_c \frac{2z-1}{z(z+1)z-3)}\,dz$, where C is the circle $|z| = 2$. **[5]**

b) Find the mechanical system which is governed by the differential equation, and with the given initial condition, **[5]**

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t}\sin t,\ x(o) = o,\ x^1(o) = 1$$

Q3) a) Find the laplace transform of the given function **[5]**

$$f(t) = 2t\sin t + \sin 2t\ u(t-\pi) + e^{-t}\sin 2t\delta(t-2).$$

b) Solve the following differential equation by series method. **[5]**

$$\frac{d^2y}{dx^2} - \frac{xdy}{dx} + x^2y = o.$$

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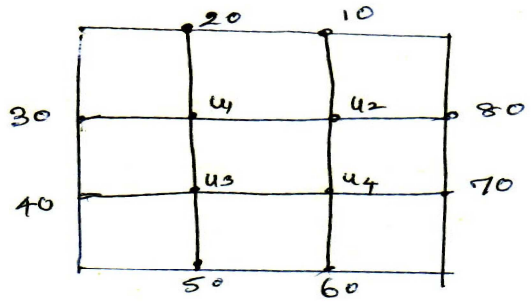
Q4) a) Calculate by power method, the largest eigen value and vector of the matrix **[5]**

$$A=\begin{bmatrix}1 & 2 & 0\\ -2 & 1 & 2\\ 1 & 3 & 1\end{bmatrix} \text{ with } X_o=\begin{bmatrix}1\\1\\1\end{bmatrix}$$

b) Solve the equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=o$ for the following square mesh with boundary values as given in the figure **[5]**



Q5) a) Solve the equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ subject to the conditions **[5]**

$$u(x,o)=\begin{cases}2x & ;\ 0\le x\le 1/2\\ 2(1-x) & ;\ 1/2\le x\le 1\end{cases} \text{ \& } u(0,t)=u(1,t)=0.$$

carry out computation for four levels by taking h = ¼ & choose k according to Bendre - schmidt method.

b) Using Rayleigh Ritz method, solve the boundry value problem $y'' - y + 4xe^x = 0$, $y(0) = y(1) = 0$. **[5]**

Q6) a) Find the function for which the functional **[5]**

$$\int_0^{\pi 12}[y^2+(y')^2-2y\sin x]dx,\ \ y(0)=0,\ y\left(\pi/2\right)=o,\ \text{can be extremized}$$

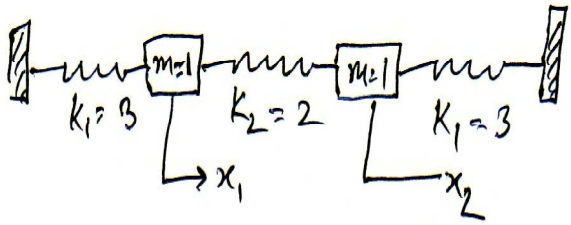
b) Solve the linear equation system using least square method **[5]**
$x - 2y = 1,\ x + y = 2,\ x + 2y = 4$

Q7) a) Show that the transformation $w = z + \frac{1}{z} - 2i$ maps the circle $|z| = 2$ onto an ellipse. **[5]**

b) Using fourier transform show that **[5]**

$$\int_0^\infty \frac{2}{1+\lambda^2}\cos\lambda x \, d\lambda = \begin{cases} 0 & ; \quad x < 0 \\ \pi & ; \quad x = 0 \\ \pi e^{-x} & ; \quad x > 0 \end{cases}$$

Q8) a) For the system shown in the figure assuming there is no friction. Find natural frequencies of the system and corresponding normal mode of the vibration using matrix method. **[5]**



b) Solve the boundary value problem $U_{tt} = U_{xx}$ with conditions $u(0,t) = u(1,t) = 0$, $u_t(x,0) = 0$ and $u(x,0) = \frac{1}{2}x(1.x)$. Find solution upto $t = 0.4$ by using $h = k = 0.1$. **[5]**