Total No. of Questions—**12**] [Total No. of Printed Pages—**8**

Seat No.	

[4757]-101

S.E. (Civil) (I Sem.) EXAMINATION, 2015

ENGINEERING MATHEMATICS—III

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

***N.B.* :—** (*i*) Attempt Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6** from Section I and Q. No. **7** *or* Q. No. **8**, Q. No. **9** *or* Q. No. **10**, Q. No. **11** *or* Q. No. **12** from Section II.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Figures to the right indicate full marks.

(*v*) Use of non-programmable electronic pocket calculator is allowed.

(*vi*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Solve the following (any *three*) : [12]

(*i*) $\left(D-1\right)^2\left(D^2+1\right)y = e^x + \sin^2\frac{x}{2}$

P.T.O.

(*ii*) $\left(D^2 + 3D + 2\right)y = xe^x \sin x$

(*iii*) $\left(D^2 - 1\right)y = e^{-x}\sin\left(e^{-x}\right) + \cos\left(e^{-x}\right)$

(By variation of parameter)

(*iv*) $\left(x^3D^3 + x^2D^2 - 2\right)y = x + \frac{1}{x^3}$ $\left(\text{where } D = \frac{d}{dx}\right)$

(*b*) Solve : [5]

$$\frac{dx}{x^2 + y^2} = \frac{dy}{2xy} = \frac{dz}{(x + y)^3 z}.$$

Or

2. (*a*) Solve the following (any *three*) : [12]

(*i*) $\left(D^3 - 3D^2 + 4D - 2\right)y = e^x + \cos x$

(*ii*) $\left(D^3 + 3D^2 - 4D - 12\right)y = 12xe^{-2x}$

(*iii*) $\left(D^3 - 6D^2 + 12D - 8\right)y = \frac{e^{2x}}{x}$

(*iv*) $(2 + 3x)^2 \frac{d^2y}{dx^2} + 3(2 + 3x)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1.$

(*b*) Solve : [5]

$$\frac{dx}{dt} = 5x + y, \frac{dy}{dt} = y - 4x.$$

3. (*a*) Find the equation of the elastic curve and its maximum deflection for the simply supported beam of length 2L, having uniformly distributed load W per unit length. [8]

(*b*) Solve the equation :

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$$

where $u(x, t)$ satisfies the following conditions :

(*i*) $u(0, t) = 0$

(*ii*) $u(L, t) = 0$ for all t

(*iii*) $u(x, 0) = x \quad 0 < x < L/2$

$= (L - x) \quad L/2 < x < L.$ [8]

Or

4. (*a*) Determine whether resonance occurs in a system consisting of a weight 32 Lb. attached to a spring with constant K = 4 Lb/ft. and external force $16 \sin 2t$ and no damping force present, initially $x = \frac{1}{2}$ and $\frac{dx}{dt} = -4$. [8]

(*b*) Solve :

$$\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2}$$

subject to the conditions :

(*i*) $u(0, t) = 0$

(*ii*) $u(5, t) = 0$

(*iii*) $u(x, 0) = 0$

(*iv*) $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 5 \sin \pi x.$ [8]

5. (*a*) Solve :

$$10x_1 - 7x_2 + 3x_3 + 5x_4 = 6$$

$$-6x_1 + 8x_2 - x_3 - 4x_4 = 5$$

$$3x_1 + x_2 + 4x_3 + 11x_4 = 2$$

$$5x_1 - 9x_2 - 2x_3 + 4x_4 = 7$$

by Gauss elimination method. [9]

(*b*) Using modified Euler's method, find $y(0.2)$ and $y(0.4)$ given $\frac{dy}{dx} = y + e^x$ $y(0) = 0$ with $h = 0.2$. [8]

Or

6. (*a*) Solve the equations :

$$10x_1 - 2x_2 - x_3 - x_4 = 3$$

$$-2x_1 + 10x_2 - x_3 - x_4 = 15$$

$$-x_1 - x_2 + 10x_3 - 2x_4 = 27$$

$$-x_1 - x_2 - 2x_3 + 10x_4 = -9$$

by Gauss-Seidel method. [8]

(*b*) Use Runge-Kutta method of fourth order solve :

$$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$$

with $y(0) = 1$ at $x = 0.2,\ 0.4$ with $h = 0.2$. [9]

SECTION II

7. (*a*) The first four moments about the working mean 28.5 of a distribution are 0.294, 7.144, 42.409 and 454.98. Calculate the moments about the mean. Also evaluate β_1, β_2. [6]

(*b*) Psychological tests of intelligence and Engineering ability were applied to 10 students. Here is a record of ungrouped data showing intelligence ratio (I.R.) and engineering ratio (E.R.). Calculate the coefficient of correlation : [6]

Student	**I.R.**	**E.R.**
1	105	101
2	104	103
3	102	100
4	101	98
5	100	95
6	99	96
7	98	104
8	96	92
9	93	97
10	92	94

(*c*) In a referendum 60% of voters voted in favour. A random sample of 200 voters was selected. What is the probability that in the sample :

(*i*) More than 130 voted in favour

(*ii*) 120 voted in favour. [5]

Or

8. (*a*) The following are scores of two batsmen A and B in a series of innings :

A	**B**
12	47
115	12
6	16
73	42
7	4
19	51
119	37
36	48
84	13
29	0

Who is better score getter and who is more consistent ? [7]

(*b*) In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. [5]

(*c*) If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals more than two will get a bad reaction. [5]

9. (*a*) A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 3$, where t is the time. Find the components of its velocity and acceleration at $t = 1$ in the direction $i + j + 3k$. [5]

(*b*) Find the directional derivative of :

$$\phi = 5x^2y - 5y^2z + 2.5z^2x$$

at the point (1, 1, 1) in the direction of the line :

$$x - 1 = 3 - y = 2z.$$ [6]

(*c*) Show that :

$$\bar{F} = (y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k$$

is both solenoidal and irrotational. [5]

Or

10. (*a*) Prove that (any *two*) :

(*i*) $\nabla\left(\nabla \cdot \frac{\bar{r}}{r}\right) = -\frac{2}{r^3}\bar{r}$

(*ii*) $\nabla \times \left[(\bar{r} \times \bar{a}) \times \bar{b}\right] = \bar{b} \times \bar{a}$

(*iii*) $\nabla \times \left[\bar{a} \times \nabla \log r\right] = \frac{2}{r^4}(\bar{a} \cdot \bar{r})\bar{r}$.

(where $\bar{a}$ and $\bar{b}$ are constant vectors.) [6]

(*b*) Determine the constants a and b such that the vector :

$$\bar{F} = (2xy + 3yz)i + (x^2 + axz - 4z^2)j + (3xy + 2byz)k$$

is irrotational. [4]

(*c*) Find the directional derivative of the divergence of :

$$F(x, y, z) = xyi + xy^2j + z^2k$$

at the point (2, 1, 2) in the direction of the outer normal to the sphere $x^2 + y^2 + z^2 = 9$. [6]

11. (*a*) If

$$\overline{F} = 2xyzi + (x^2z + 2y)j + x^2yk,$$

then find the work done in moving a particle under this force field from (0, 1, 1) to (1, 2, 0). [5]

(*b*) Using Stokes' theorem, evaluate :

$$\int_C (xy\,dx + xy^2\,dy)$$

where C is the square with vertices (1, 0), (–1, 0), (0, 1), (0, –1). [6]

(*c*) Evaluate :

$$\iint_S (2xy\,i + yz^2\,j + 2z\,k) \cdot d\overline{S}$$

where S is the total surface of a region bounded by $x = 0$, $y = 0$, $z = 0$, $y = 3$ and $x + 2z = 6$. [6]

Or

12. (*a*) Evaluate :

$$\oint_C \left[(x^2 + 2y)dx + (4x + y^2)dy\right]$$

by Green's theorem, where C is the boundary of the region bounded by $y = 0$, $y = 2x$ and $x + y = 3$. [5]

(*b*) Using Stokes' theorem show that :

$$\oint_C (y\,dx + z\,dy + x\,dz) = -\iint_S (\cos\alpha + \cos\beta + \cos\gamma)\,dS$$

where α, β, γ are angle made by the normal to the surface S, with x, y, z axes respectively. [6]

(*c*) Show that the velocity potential :

$$\phi = \frac{a}{2}(x^2 + y^2 - 2z^2)$$

satisfies the Laplace's equation. Also determine the stream-lines. [6]