Total No. of Questions—**12**] [Total No. of Printed Pages—**8+1**

Seat No.	

[4757]-190

S.E. (I Sem.) EXAMINATION, 2015

(Common to Computer Engineering and IT)

DISCRETE STRUCTURES

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

N.B. :— (*i*) Answers to the two Sections should be written in separate answer-books.

(*ii*) In Section I attempt Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**.

(*iii*) In Section II attempt Q. No. **7** *or* Q. No. **8**, Q. No. **9** *or* Q. No. **10**, Q. No. **11** *or* Q. No. **12**.

(*iv*) Neat diagrams must be drawn wherever necessary.

(*v*) Figures to the right indicate full marks.

(*vi*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Determine whether each of the following statements is true or false. Briefly explain your answer : [8]

(*i*) $A \cup P(A) = P(A)$

(*ii*) $\{A\} \cup P(A) = P(A)$

(*iii*) $\{A\} \cap P(A) = A$

(*iv*) $A - P(A) = A$.

P.T.O.

(*b*) Obtain the conjunctive normal form of : [8]

(*i*) $(\sim p \rightarrow r) \wedge (p \leftrightarrow q)$

(*ii*) $(p \wedge q) \vee (\sim p \wedge q \wedge r)$.

Or

2. (*a*) Give truth table and comment whether tautology or contradiction : [8]

(*i*) $\neg(\neg P \vee \neg Q)$

(*ii*) $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \vee R)$

(*iii*) $(P \rightarrow Q) \wedge (Q \rightarrow P)$.

(*b*) To describe various restaurants in the city, we let p denotes the statement "The food is good", q denotes the statement "The service is good", r denotes the statement "The rating is three-star". Write the following statement is symbolic form : [6]

(*i*) Either the food is good or the service is good or both.

(*ii*) Either the food is good or the service is good but not both.

(*iii*) The food is good while the service is poor.

(*iv*) It is not the case that both the food is good and the rating is three-star.

(*v*) If both the food and services are good, then the rating will be three-star.

(*vi*) It is not true that a three-star rating always means good food and good service.

(*c*) Define the term countably infinite set and uncountably infinite set. [2]

3. (*a*) Define with examples : [8]

(*i*) Subgroup

(*ii*) Cyclic Group

(*iii*) Field

(*iv*) Integral domain.

(*b*) Prove the following results for the group G : [6]

(*i*) The identity element is unique.

(*ii*) Each a in G has a unique inverse a^{-1}.

(*iii*) $ab = ac$ implies $b = c$.

(*c*) What is homomorphism and automorphism in an algebraic system ? [2]

Or

4. (*a*) A central groupoid is an algebraic system (A, *), where * is a binary operation such that $(a * b) * (b * c) = b$. For all a, b, c belongs to A. Such that : [8]

(*i*) $a * ((a * b) * c) = a * b$

(*ii*) $(a * (b * c)) * c = b * c$.

(*b*) Let Z = {0, 1, 2, ..., $n - 1$}. Let $\Diamond$ be a binary operation such that $a \Diamond b$ = remainder of $a.b$ divided by n. Construct a table of $n = 4$. Is $(Z_4, \Diamond)$ a groupoid, monoid, semi-group and abelian group. [8]

5. (*a*) Let R be a binary relation on the set of all strings of 0's and 1's such that R = {(*a*, *b*) | *a* and *b* are strings that have the same number of 0's} : [8]

(*i*) Is R reflexive ?

(*ii*) Is R symmetric ?

(*iii*) Is R antisymmetric ?

(*iv*) Is R transitive ?

(*v*) Is R equivalence relation ?

(*vi*) Is R a partial ordering relation ?

(*b*) Find the transitive closure of R by Warshall's algorithm : A = {set of integers <=10}, B = {(*a*, *b*) | *a* divides *b*}. [8]

(*c*) Let X = {1, 2, 3} and *f* and *g*, be function from X to X given by : [2]

f = {(1, 2), (2, 3), (3, 1)}, *g* = {(1, 2), (2, 1), (3, 3)}.

Find :

(*i*) *f* o *g*

(*ii*) *g* o *f*.

Or

6. (*a*) For a given set A consider R = {(*x*, *y*) | *x* ∈ P(A), *y* ∈ P(A), *x* Ì *y*}. Show that R is a partial ordering relation. What is the length of the longest chain in the poset {P(A), R} ? [8]

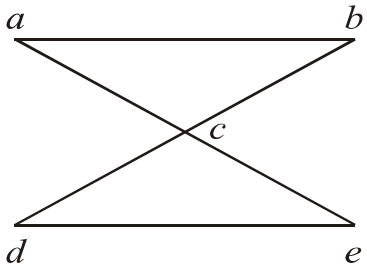
(*b*) Let A = {1, 2, 3, 4, 6, 12, 16, 36} and let R be the relation on A defined by *x* divides *y*. Draw the Hasse diagram. Compare it with digraph. Determine whether R is a reflexive, transitive and symmetric. [8]

(*c*) Let A = {2, 3, 4, 6} and let *a*R*b* if *a* divides *b*. State R is a partial order. [2]

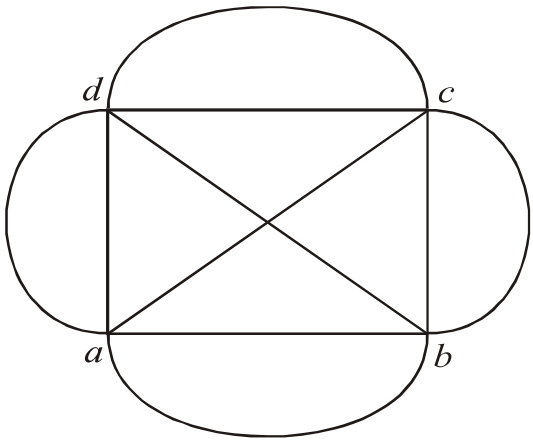
SECTION II

7. (*a*) Which of the following graphs have a Euler circuit or path or Hamiltonian cycle ? Write the path or circuit : [6]

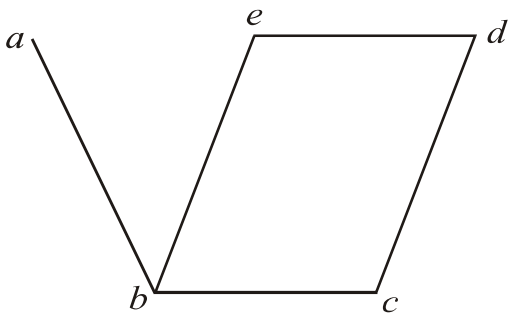
(*i*)



(*ii*)



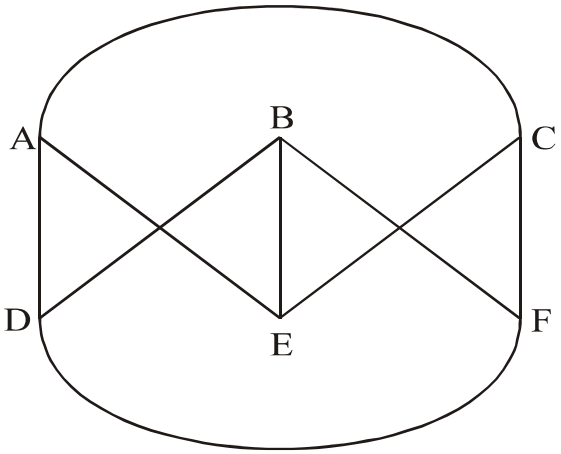
(*iii*)



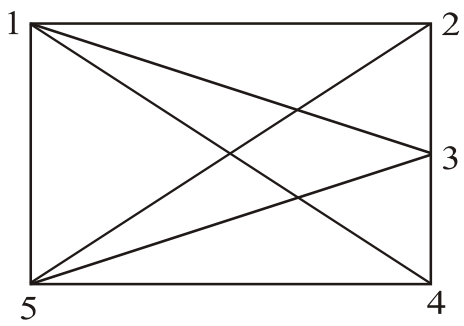
(*b*) Define graph and multigraph. Give any *two* applications of graph and represent them in graph notation. [6]

(*c*) Identify whether the graphs given are planar or not. Draw planar representation if possible : [6]

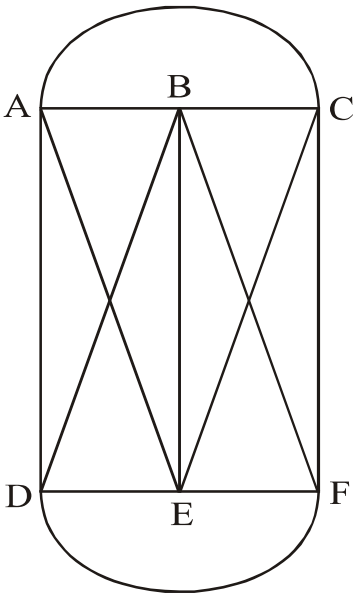
(*i*)



(*ii*)



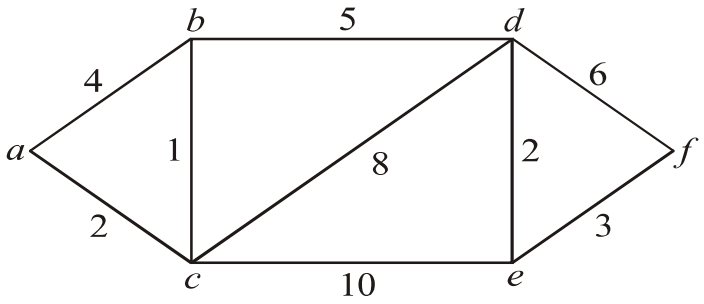
(*iii*)



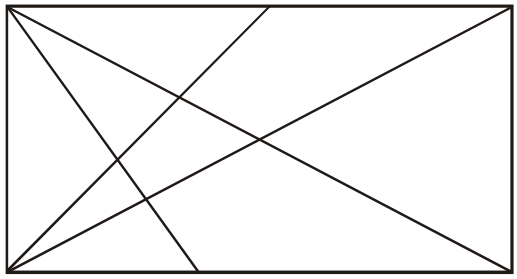
Or

8. (*a*) State and prove Euler's formula for a connected planar graph of order n, size e and with f faces. [6]

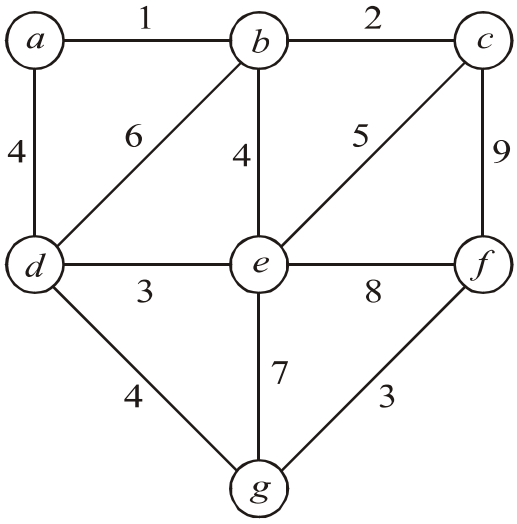
(*b*) Use Dijkstra algorithm to find the shortest path from a to f. [8]



(*c*) Draw isomorphic graph of a graph shown in the following figure but no crossover of edges. [4]



9. (*a*) Using Prim's algorithm construct minimal spanning tree starting at vertex a. [6]



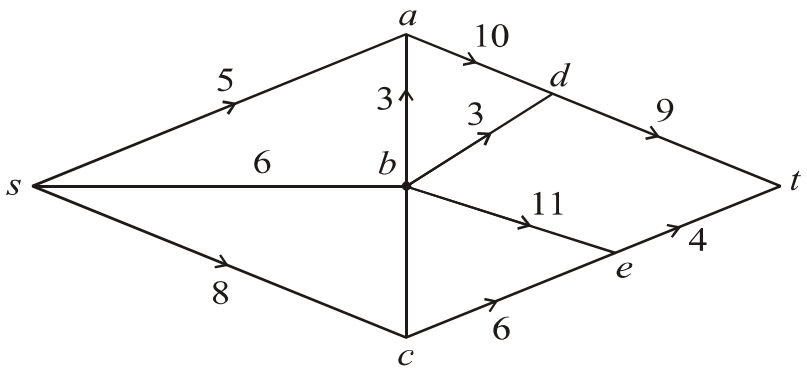
(*b*) Draw a binary search tree for input data 200, 100, 300, 50, 150, 250, 400, 10, 75, 125, 175. Which is a root, leaf nodes and interior nodes. [6]

(*c*) Construct the expression tree for the following expression : (3 – (2(–11 – (9 – 4)))) ÷ (2 + (3 + (4 + 7))).

Also evaluate expression. [4]

Or

10. (*a*) Use labeling procedure to find maximum flow in transport network shown below. Define corresponding minimal cuts. [8]



(*b*) Use Huffman coding to encode the following symbol with the frequencies listed A : 0.08, B : 0.010, C : 0.12, D : 0.15, E : 0.20, F : 0.35. What is average number of bits used to encode the character ? [8]

11. (*a*) If repetitions are not permitted, how many four digit numbers can be formed from digits 1, 2, 3, 7, 8 and 5. [6]

(*b*) Show that : [6]

$$c(2n,\ 2) = 2c(n,\ 2) + n^2.$$

(*c*) How many seven letter words can be formed using the letters of the word BENZENE ? [4]

Or

12. (*a*) Three students A, B and C are swimming in the race. A and B have same probability of winning and each is twice as likely to win as C. Find probability that : [6]

(*i*) B wins

(*ii*) C wins

(*iii*) B or C wins.

(*b*) A woman has 11 friends of them six are women : [6]

(*i*) In how many ways can she invite three or more ?

(*ii*) In how many ways can she invite three or more of them if she wants same number of men and women (including herself) ?

(*c*) A coin is tossed 3 times. Find the probability that there will appear : [4]

(*i*) Three heads

(*ii*) Exactly 2 heads

(*iii*) No heads.