Total No. of Questions—**12**] [Total No. of Printed Pages—**8**

Seat No.	

[4757]-131

S.E. (Electrical/Inst./Comp./I.T.) (First Semester)

EXAMINATION, 2015

ENGINEERING MATHEMATICS–III

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

N.B. **:—** (*i*) Answer Q. No. **1** or Q. No. **2**, Q. No. **3** or Q. No. 4, Q. No. **5** or Q. No. **6** from Section I and Q. No. **7** or Q. No. 8, Q. No. **9** or Q. No. **10**, Q. No. **11** or Q. No. **12** from Section II.

(*ii*) Answers to the two sections should be written in separate answer-books.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of logarithmic tables, electronic pocket calculator is allowed.

(*v*) Neat diagrams must be drawn wherever necessary.

(*vi*) Assume suitable data, if necessary.

SECTION I

1. (A) Solve the following (any *three*) : [12]

(*i*) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$

(*ii*) $(D^2 - 4)y = e^{3x}\ x^2$

(*iii*) $\dfrac{d^2y}{dx^2} + y = \operatorname{cosec} x$ (by method of variation of parameters).

(*iv*) $x^2\dfrac{d^2y}{dx^2} - 3x\dfrac{dy}{dx} + 5y = x^2 \sin(\log x)$.

(B) An uncharged condensor of capacity C charged by applying an e.m.f. of value $E \sin \dfrac{t}{\sqrt{LC}}$ through the leads of inductance L and of negligible resistance. The charge Q on the plate of condenser satisfies the differential equation :

$$\frac{d^2Q}{dt^2} + \frac{Q}{LC} = \frac{E}{L}\sin\frac{t}{\sqrt{LC}}.$$

Find charge Q at any time *t*. [5]

Or

2. (A) Solve the following (any *three*) : [12]

(*i*) $(D^2 - 4D + 4)y = x\,e^{2x}\ \sin 2x$

(*ii*) $(D^3 + D)y = \cos x$

(*iii*) $\dfrac{dx}{y} = \dfrac{dy}{-x} = \dfrac{dz}{xe^{x^2+y^2}}$

(*iv*) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + y = 5^x + 2 + e^x$.

(B) Solve the system of equations : [5]

$$\frac{dx}{dt} + y = e^{t}; \quad \frac{dy}{dt} + x = e^{-t}.$$

3. (A) Find the analytic function $f(z)$ whose imaginary part is : [6]

$$V = \cos x \cosh y.$$

(B) Evaluate :

$$\oint_C \frac{\sin^2 z}{\left(z - \frac{\pi}{6}\right)^3} dz$$

where C is the circle $|z| = 1$. [5]

(C) Find the map of straight line $y = x$ under the transformation $w = \frac{z-1}{z+1}$. [5]

Or

4. (A) If $f(z)$ is analytic, show that : [5]

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4|f'(z)|^2.$$

(B) By using Cauchy's residue theorem, evaluate : [6]

$$\oint_C \frac{z+2}{z^2+1} dz$$

where $C : |z - i| = \frac{1}{2}$.

(C) Find the bilinear transformation which maps the points $-i$, 0, $2 + i$ of z-plane to the points 0, $-2i$, 4. [5]

5. (A) Using Fourier integral representation, prove that : [6]

$$\int_0^\infty \frac{1-\cos\lambda\pi}{\lambda}\sin\lambda x\, d\lambda = \begin{cases} \frac{\pi}{2}, & 0 < x < \pi \\ 0, & x > \pi \end{cases}.$$

(B) Find Fourier cosine integral representation of : [5]

$$f(x) = 2e^{-5x} + 5e^{-2x}.$$

(C) Find z-transform of the following (any *two*) : [6]

(*i*) $f(k) = k\,2^k, \quad k \geq 0$

(*ii*) $f(k) = \sin(2k+5), \quad k \geq 0$

(*iii*) $f(k) = \dfrac{3^k}{k}, \quad k \geq 1$.

Or

6. (A) Find inverse z-transform of the following (any *two*) : [6]

(*i*) $F(z) = \dfrac{1}{(z-3)(z-2)}, \quad |z| < 2$

(*ii*) $F(z) = \dfrac{z^2}{\left(z-\frac{1}{4}\right)\left(z-\frac{1}{5}\right)}, \quad |z| > \dfrac{1}{5}$

(*iii*) $F(z) = \dfrac{10z}{(z-1)(z-2)}$ (by inversion integral method).

(B) Solve the difference equation : [5]

$$f(k+1) - 2f(k) = 2^k,\ f(0) = 1,\ k \geq 0.$$

(C) Using inverse sine transform, find $f(x)$ if : [6]

$$F_s(\lambda) = \frac{1}{\lambda}e^{-a\lambda}.$$

SECTION II

7. (A) The first four moments of a distribution about the value 4 of the variable are –1.5, 17, –30 and 108. Find the moments about mean, coefficient of skewness and kurtosis. [7]

(B) Find the correlation coefficient and regression lines for the data : [9]

x	y
1	2
2	5
3	3
4	8
5	7

Or

8. (A) In sampling a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2. Out of 1000 such samples, how many would be expected to contain at least 3 defective parts. [6]

(B) In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and standard deviation of 60 hours. Estimate the number of bulbs likely to burn for more than 1920 hours and less than 2160 hours.

(Area corresponding to $z = 2$ is 0.4772). [6]

(C) In a Poisson distribution, if :

$$p(r = 1) = 4\, p(r = 2),$$

find $p\,(r = 3)$. [4]

9. (A) Find the angle between the surfaces :

$$x^2 + y^2 + z^2 = 9 \quad \text{and} \quad z = x^2 + y^2 - 3$$

at the point (2, –1, 2). [4]

(B) Find the directional derivative of $\phi = 5x^2y - 5y^2z + z^2x$ at the point (1, 1, 1) in the direction of the line : [5]

$$\frac{x-1}{2} = \frac{y-3}{-2} = z\,.$$

(C) Prove that (any *two*) : [8]

(*i*) $\nabla\left(\frac{\bar{a}\,.\,\bar{r}}{r^5}\right) = \frac{\bar{a}}{r^5} - \frac{5\,(\bar{a}\,.\,\bar{r})}{r^7}\,\bar{r}$

(*ii*) $\nabla^2\left[\nabla\,.\left(\frac{\bar{a}}{r^2}\right)\right] = \frac{2}{r^4}$

(*iii*) $\nabla^4\,(r^2 \log r) = \frac{6}{r^2}\,.$

Or

10. (A) If a vector field is given by : [6]

$$\overline{\mathrm{F}} = (x^2 - y^2 + x)\,\bar{i} - (2xy + y)\,\bar{j}\,,$$

is this field irrotational ? If so find its scalar potential.

(B) Find the values of a and b such that the surfaces : [6]

$$ax^2 - byz = (a+2)x \quad \text{and} \quad 4x^2y + z^3 = 4,$$

cut orthogonally at (1, –1, 2).

(C) Show that :

$$\overline{F} = f(r)\,\overline{r},$$

is always irrotational and find $f(r)$ such that $\overline{F}$ is solenoidal. [5]

11. (A) Evaluate :

$$\int_C \overline{F} \cdot d\overline{r}$$

along the path C is $x = 2t$, $y = t$, $z = t^3$ from $t = 0$ to $t = 1$.

Where $\overline{F} = (2y+3)\overline{i} + xz\,\overline{j} + (yz - x)\overline{k}$. [5]

(B) Evaluate using Stokes theorem : [6]

$$\int_C (2x - y)\,dx - yz^2 dy - y^2 z dz,$$

where C is the circle $x^2 + y^2 = 1,\ z = 0$.

(C) Evaluate :

$$\iint_S \overline{F} \cdot \hat{n}\, dS, \quad \text{where} \quad \overline{F} = 4xz\,\overline{i} - y^2\,\overline{j} + yz\,\overline{k}$$

and S is the surface of the cube bounded by :

$x = 0,\ x = 1,\ y = 0,\ y = 1,\ z = 0,\ z = 1.$ [6]

Or

12. (A) Apply Green's theorem to evaluate :

$$\int_C (2x^2 - y)\, dx + (x + y^2)\, dy$$

where C is the boundary of the region defined by :

$$y = \sqrt{x} \quad \text{and} \quad y = x^2. \qquad [5]$$

(B) Evaluate :

$$\iint_S \nabla \times \overline{F} \,.\, d\overline{S}$$

for $\overline{F} = y\,\overline{i} + z\,\overline{j} + 2x^2\,\overline{k}$,

where S is the surface of the paraboloid $z = 16 - x^2 - y^2,\ z \geq 0$. [6]

(C) Evaluate :

$$\iint_S 2(x^3\,\overline{i} + y^3\,\overline{j} + z^3\,\overline{k}) \,.\, d\overline{S},$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$. [6]