Total No. of Questions—12] [Total No. of Printed Pages—8

Seat No.	

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S.E. (Mech./Auto) (First Semester) EXAMINATION, 2015

ENGINEERING MATHEMATICS–III

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

N.B. :— (*i*) Answer *three* questions from Section I and *three* questions from Section II.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.

(*v*) Assume suitable data if necessary.

SECTION I

1. (*a*) Solve the following DE (any *three*) : [12]

(*i*) $(D - 2)^2 (D + 1)y = e^{2x} + 2^{-x}$

(*ii*) $(D^2 - 6D + 9)y = \dfrac{e^{3x}}{x^2}$ (by variation of parameters)

(*iii*) $(1 + x)^2 \dfrac{d^2y}{dx^2} + (1 + x)\dfrac{dy}{dx} + y = 2\sin [\log (1 + x)]$

(*iv*) $(D^2 - 2D + 5)y = 25x^2$.

(*b*) Solve the simultaneous linear DE with given conditions :

$$\frac{du}{dx} + v = \sin x, \quad \frac{dv}{dx} + u = \cos x$$

given that $x = 0$, when $u = 1$ and $v = 0$. [5]

P.T.O.

Or

2. (*a*) Solve the following DE (any *three*) : [12]

(1) $(D^3 + D)y = \cos x$

(2) $x^2 \frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = x^5$

(3) $(D^2 + 2D + 1)y = e^{-x} \log x$

(4) $\frac{dx}{3z - 4y} = \frac{dy}{4x - 2z} = \frac{dz}{2y - 3x}$.

(*b*) A body of weight W = 1N suspended from a spring stretches it 4 cm. If the weight is pulled down 8 cm below equilibrium position and then released :

(*i*) Set up a differential equation

(*ii*) Find position and velocity as a function of time. [5]

3. (*a*) Find the Laplace transform of (any *two*) : [6]

(*i*) $\int_0^t e^u u^3 \, du$

(*ii*) $t^2 \sin 2t$

(*iii*) $\frac{1 - e^{-t}}{t}$.

(*b*) Find the Inverse Laplace Transform of (any *two*) : [6]

(*i*) $\tan^{-1} \frac{1}{s}$

(*ii*) $\frac{1}{s^2(s + 1)}$

(*iii*) $\frac{s}{(s - 1)(s - 2)(s - 3)}$.

(*c*) Solve the integral equation :

$$\int_0^{\infty} f(x) \sin \lambda x \, dx = \begin{cases} 1 - \lambda, & 0 \le \lambda \le 1 \\ 0, & \lambda \ge 1. \end{cases}$$ [5]

Or

4. (*a*) Find the Fourier sine transform of $e^{-|x|}$. Hence evaluate

$$\int_0^{\infty} \frac{x \sin mx}{1 + x^2} dx.$$ [6]

(*b*) Using Fourier Integral representation, show that :

$$\int_0^{\infty} \frac{1 - \cos \lambda\pi}{\lambda} \sin \lambda x \, d\lambda = \begin{cases} \frac{\pi}{2}, & 0 < x < \pi \\ 0, & x > \lambda. \end{cases}$$ [6]

(*c*) Solve by Laplace Transform method :

$y'' - 3y' + 2y = 12e^{-2t}$, $y(0) = 2$, $y'(0) = 6$. [5]

5. (*a*) Solve $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, if :

(*i*) u is finite for all t

(*ii*) $u(0, t) = 0$, for all t

(*iii*) $u(l, t) = 0$, for all t

(*iv*) $u(x, 0) = u_0$, for $\le x \le l$

l being length of the bar. [8]

(*b*) A string is stretched and fastened to two points 'l' apart. Motion is started by displacing the string in the form $u = a \sin \frac{\pi x}{l}$ from which it is released at time $t = 0$. Find the displacement $u(x, t)$ from one end. [8]

Or

6. (*a*) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to :

(*i*) $u(x, 0) = 0$

(*ii*) $u(x, l) = 0$

(*iii*) $u(\infty, y) = 0$

(*iv*) $u(0, y) = u_0$. [8]

(*b*) The initial temperature along the length of an infinite bar is given by

$$u(x, 0) = \begin{cases} 2 & |x| < 1 \\ 0, & |x| > 1. \end{cases}$$

If the temperature $u(x, t)$ satisfies the equation :

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, -\infty < x < \infty, \quad t > 0.$$

Find the temperature at any point of the bar at any time. [Use Fourier Transform method]. [8]

SECTION II

7. (*a*) The marks obtained by a class of students in Economics and Statistics. Calculate coefficient of correlation : [6]

Marks in Economics	Marks in Statistics
23	25
28	22
42	38
17	21
26	27
35	39
29	24
37	32
16	18
46	44

(*b*) A manufacturer of cotter pins knows that 3% of his product is defective. If he sells cotter pins in boxes of 100 pins will be defective in a box. Find probability that a box will fail to meet the guaranted quality. [5]

(*c*) Find the first four moments about the mean for the data : [6]

Marks	**No. of Students**
0–10	1
10–20	6
20–30	10
30–40	15
40–50	11
50–60	7

8. (*a*) The following runs scored by two Batsman A and B in ten Matches :

A	**B**
30	34
44	46
66	70
62	38
60	55
34	48
80	60
46	34
20	45
38	30

Determine, who is more consistent ? [6]

(*b*) In a test on 2000 electric bulbs it was found that the life of a particular make was normally distributed with an average time 2040 hours and S.D. of 60 hours. Estimate the number of bulbs likely burn for more than 1920 hours but less than 2160 hrs.

(Given : $z = 2$, Area = 0.4772). [6]

(*c*) The number of Computer Books borrowed from a library during a particular week is given below :

Days	**No. of Books**
Mon.	140
Tues.	132
Wed.	160
Thur.	148
Fri.	134
Sat.	150

Test the hypothesis that the no. of books borrowed does not depend on the day of week. (Given : $\chi^2_{0.05}$ = (11.07). [5]

9. (*a*) Show that tangent at any point on the curve $x = e^t \cos t$, $y = e^t \sin t$, $z = e^t$ at $t = 0$ makes constant angle with z-axis. [5]

(*b*) Find the directional derivative of $\phi = xy^2 + yz^3$ at (1, –1, 1) in the direction normal to the surface $x^2 + y^2 + z^2 = 9$ at (1, 2, 2). [5]

(*c*) Prove that (any *two*) : [6]

(*i*) $\nabla^2 \left\{ \nabla \cdot \frac{\overline{r}}{r^2} \right\} = \frac{2}{4}$

(*ii*) $\nabla \left(\frac{\overline{a} \times \overline{r}}{r^n} \right) = \frac{(2-n)\overline{a}}{r^n} + \frac{n(\overline{a} \cdot \overline{r})}{r^{n+2}} \overline{r}$

(*iii*) $\nabla[\overline{a} \cdot (\overline{r} \times \overline{b})] = \overline{b} \times \overline{a}$.

Or

10. (*a*) Show that :

$$\overline{\mathrm{F}} = (y \sin z - \sin x)\overline{i} + (x \sin z + 2yz)\overline{j} + (xy \cos z + y^2)\overline{k}$$

is irrotational. Also find scalar point function 'ϕ' such that $\overline{\mathrm{F}} = \nabla\phi$. [5]

(*b*) Show that $f(r)\overline{r}$ is irrotational. Determine $f(r)$ such that the field is solenoidal. [6]

(*c*) If the directional derivative of $\phi = axy + byz + czx$ at (1, 1, 1) has the magnitude 4 in the direction parallel to x-axis. Find the values of a, b, c. [5]

11. (*a*) Evaluate $\int_{\mathrm{C}} \overline{\mathrm{F}} \cdot d\overline{r}$

for $\overline{\mathrm{F}} = 3x^2\overline{i} + (2xz - y)\overline{j} + z\overline{k}$

along the curve $\overline{r} = t^2\overline{i} + 2t\overline{j} + (2t^2 - 1)\overline{k}$ from $t = 0$ to $t = 1$. [5]

(*b*) By using Green's theorem, evaluate

$\int_{\mathrm{C}} \overline{\mathrm{F}} \cdot d\overline{r}$ for $\overline{\mathrm{F}} = (\sin y - y^3)\overline{i} + (xy^2 + x \cos y)\overline{j}$

where 'C' is circle $x^2 + y^2 = a^2$. [6]

(*c*) Use divergence theorem to evaluate $\iint_S \overline{F} \cdot d\overline{s}$, where S is the surface of cone $z^2 = x^2 + y^2$ above XoY plane and bounded by the plane $z = 4$, where $\overline{F} = 4zx\overline{i} + xyz^2\overline{j} + 3z\overline{k}$. [6]

Or

12. (*a*) Verify Stokes' theorem for the field :

$$\overline{F} = (2x - y)\overline{i} - yz^2\overline{j} - y^2z\overline{k}$$

over the surface of hemisphere $x^2 + y^2 + z^2 = 1$ above XoY plane. [6]

(*b*) Evaluate $\iint_S \overline{F} \cdot d\overline{s}$

for $\overline{F} = 4x\overline{i} - 2y^2\overline{j} + z^2\overline{k}$

and S is the surface of cylinder $x^2 + y^2 = 4$, $z = 0$, $z = 3$. [6]

(*c*) Find the work done in moving a particle once round the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$, $z = 0$ under the force field

$$\overline{F} = (2x - y + z)\overline{i} + (x + y - z^2)\overline{j} + (3x - 2y + 4z)\overline{k}.$$ [5]