| Seat No. | |

[4757]-1001

S.E. (Civil) (First Semester) EXAMINATION, 2015

ENGINEERING MATHEMATICS III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

N.B. :— (*i*) Neat diagrams must be drawn wherever necessary.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use of electronic pocket calculator and steam tables is allowed.

(*iv*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* of the following : [8]

(*i*) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + y = 4 + 2^x + 3e^{-x} + \cos x$

(*ii*) $x^3 \dfrac{d^3y}{dx^3} + 2x^2 \dfrac{d^2y}{dx^2} + 2y = x + x^{-1}$

(*iii*) Use the method of variation of parameters to solve the linear differential equation :

$$\dfrac{d^2y}{dx^2} + y = \operatorname{cosec} x .$$

(*b*)   Solve the following system of linear equations by Gauss Elimination method :   [4]

$$5x - 2y + 2z = 5, \quad 2x + y - z = 2, \quad x - y + z = 1.$$

Or

2.   (*a*)   Solve the system of simultaneous symmetric equations :   [4]

$$\frac{dx}{2x} = \frac{dy}{-y} = \frac{dz}{4xy^2 - 2z}.$$

(*b*)   Apply Runge-Kutta method of 4th order to solve the differential equation :

$$\frac{dy}{dx} = x + y^2, \qquad y(0) = 1$$

to find y for $0 \leq x \leq 0.2$ with $h = 0.1$.   [4]

(*c*)   Solve the following system of equations by Cholesky method :   [4]

$$2x + 3y + z = 0$$

$$x + 2y - z = -2$$

$$-x + y + 2z = 0.$$

3.   (*a*)   The first four moments about the value 4 are −1.5, 17, −30 and 108. Calculate the moments about the mean. Also find coefficient of skewness and kurtosis.   [4]

(b)   Assume the mean height of soldiers to be 68.22 inches with a variance of 10.8 inches. How many soldiers in a regiment of 10,000 would you expect to be over 6 feet tall, where the data is normally distributed. (Given : $\phi(1.15) = 0.3749$)   [4]

(c)   Find the directional derivative of

$$\phi = xy^2 + yz^3$$

at the point (2, −1, 1) in the direction of vector $i + 2j + 2k$.   [4]

Or

4.   *(a)*   Attempt any *one* :   [4]

(i)   Prove that $\dfrac{\overline{r}}{r^3}$ is solenoidal.

(ii)   Show that :

$$\nabla \cdot \left[r \, \nabla \frac{1}{r^n} \right] = \frac{n(n-2)}{r^{n+2}}.$$

(b)   Verify whether :

$$\overline{F} = (2xyz^2)i + (x^2z^2 + z\cos yz)j + (2x^2yz + y\cos yz)k$$

is irrotational.   [4]

(c)   Two lines of regression are :

$$5y - 8x + 17 = 0 \quad \text{and} \quad 2y - 5x + 14 = 0.$$

If $\sigma_y^2 = 16$, find :   [4]

(i) σ_x^2

(ii)   Coefficient of correlation.

5.   (a)   Evaluate $\int\limits_C \overline{F}.d\overline{r}$ for

$$\overline{F} = (2xy + 3x^2)\overline{i} + (x^2 + 4yz)\overline{j} + (2y^2 + 6yz)\overline{k}$$

where C is the curve $x = t$, $y = t^2$, $z = t^3$ joining (0, 0, 0) and (1, 1, 1).   [4]

(b)   Use divergence theorem to evaluate $\iint\limits_S \overline{F}.d\overline{s}$

for $\overline{F} = 4xz\,\overline{i} - y^2\,\overline{j} + yz\,\overline{k}$ over the surface of cube bounded by the planes $x = 0$, $x = 2$, $y = 0$, $y = 2$, $z = 0$, $z = 2$.   [4]

(c)   Using Stokes' theorem evaluate $\iint\limits_S (\nabla \times \overline{F}).d\overline{s}$

for $\overline{F} = (x^3 - y^3)\overline{i} - xyz\,\overline{j} + y^3\,\overline{k}$ where S is the surface $x^2 + ay^2 + z^2 - 2x = 4$ above the plane $x = 0$.   [5]

Or

6. (*a*) Using Green's theorem, evaluate $\oint \bar{F} \cdot d\bar{r}$ for the field :

$$\bar{F} = x^2\,\bar{i} + xy\,\bar{j}$$

over the region R enclosed by $y = x^2$ and then line $y = x$. [4]

(*b*) Use divergence theorem to evaluate :

$$\iint\limits_{S} (x^3\,\bar{i} + y^3\,\bar{j} + z^3\,\bar{k}) \cdot d\bar{s}$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = 16$. [4]

(*c*) Evaluate $\int\limits_{C} \bar{F} \cdot d\bar{r}$ using Stokes' theorem for :

$$\bar{F} = 4y\bar{i} + 2z\,\bar{j} + 6y\bar{k}$$

where C is the intersection of :

$$x^2 + y^2 + z^2 = 2z, \quad x = z - 1.$$ [5]

7. (*a*) A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y = a\sin\dfrac{\pi x}{l}$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by : [7]

$$y(x,\, t) = a\sin\left(\frac{\pi x}{l}\right)\cos\left(\frac{\pi ct}{l}\right).$$

(*b*)   Solve :

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$$

subject to :   [6]

(*i*) $u(0, t) = 0$

(*ii*) $u_x(l, t) = 0$

(*iii*) $u(x, t)$ is bounded and

(*iv*) $u(x, 0) = \frac{u_0 x}{l}, \; 0 \le x \le l$.

Or

8.   (*a*)   Solve the equation :

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$$

subject to :   [7]

(*i*) $v = 0$ when $y \to \infty$ for all x

(*ii*) $v = 0$ when $x = 0$ for all y

(*iii*) $v = 0$ when $x = l$ for all y

(*iv*) $v = x(l - x)$ when $y = 0$ for $0 < x < l$ T.

(*b*) Solve the wave equation :   [6]

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

under the conditions :

(*i*) $u(0, t) = 0$

(*ii*) $u(\pi, t) = 0$

(*iii*) $\left(\dfrac{\partial u}{\partial t} \right)_{t=0} = 0$

(*iv*) $u(x, 0) = x, \quad 0 < x < \pi$.