Total No. of Questions—8] [Total No. of Printed Pages—8

Seat No.	

[4757]-1011

S.E. (Mech./Automobile) (First Semester)

EXAMINATION, 2015

ENGINEERING MATHEMATICS—III

(2012 COURSE)

Time : Two Hours **Maximum Marks : 50**

N.B. **:—** (*i*) Attempt Q. No. **1** *or* Q. No. **2,** Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6,** Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* of the following : [8]

(*i*) $(D^2 + 9)\ y = x.\sin\ 2x$

(*ii*) $(D^2 - 6D + 13)\ y = e^{3x}.\sin\ 4x + 3^x$

(*iii*) $(D^2 + 4)\ y = \dfrac{1}{1 + \sin 2x}$,

(using method of variation of parameters.)

P.T.O.

(*b*) Using Fourier cosine integral of e^{-mx} $(m > 0)$ show that : [4]

$$\int_0^\infty \frac{m \cos \lambda x \, d\lambda}{m^2 + \lambda^2} = \frac{\pi}{2} . e^{-mx}, \ (m > 0, \ x > 0).$$

Or

2. (*a*) A body of weight "W = 20 N" is hung from a spring. A pull of "40 N" will stretch the spring to 10 cm. The body is pulled down to 20 cm below the static equilibrium position and then released. Find the displacement of the body from its equilibrium position in time '*t*' seconds. [4]

(*b*) Solve any *one* of the following : [4]

(*i*) Find the Laplace transform of

$$f(t) = \int_0^t \frac{\sin t}{t} \, dt.$$

(*ii*) Find the inverse Laplace transform of

$$\mathrm{F}(s) = \log\left(\frac{s + 4}{s + 8}\right).$$

(*c*) Solve the following differential equation by using Laplace transform method : [4]

$$y''(t) + 9y(t) = 18t,$$

given that :

$$y(0) = 0, \quad y\left(\frac{\pi}{2}\right) = 1.$$

3. (*a*) Calculate the first four moments about the mean of the given distribution : [4]

x	***f***
2.0	4
2.5	36
3.0	60
3.5	90
4.0	70
4.5	40
5	10

(*b*) Between 2 p.m. and 3 p.m. the average number of phone calls per minute coming into company are 3. Find the probability that during one particular minute there will be 2 or less calls. [4]

(*c*) Find the directional derivative of

$$\phi = xy^2 + yz^3$$

at (1, –1, 1) in the direction of tangent to the curve $x = \sin t, y = \cos t, z = t$ at $t = \frac{\pi}{4}$. [4]

Or

4. (*a*) Find the coefficient of correlation for the following data : [4]

x	***y***
10	18
14	12
18	24
22	06
26	30
30	36

(*b*) Prove that (any *one*) : [4]

(*i*) $\nabla\left(\frac{\bar{a} \cdot \bar{r}}{r^n}\right) = \frac{\bar{a}}{r^n} - \frac{n(\bar{a} \cdot \bar{r})}{r^{n+2}}\bar{r}$

(*ii*) $\nabla^4\left(r^2 \log r\right) = \frac{6}{r^2}$.

(*c*) Show that : [4]

$$\bar{F} = \left(6xy + z^3\right) i + \left(3x^2 - z\right) j + \left(3xz^2 - y\right)k$$

is irrotational. Find the scalar ϕ such that :

$$\bar{F} = \nabla\phi.$$

5. (*a*) Evaluate :

$$\int_C \overline{F} \cdot d\overline{r}$$

for

$$\overline{F} = 3x^2\overline{i} + (2xz - y)\overline{j} + z\overline{k}$$

along the curve :

$$x = 2t^2, \; y = t, \; z = 4t^2 - t$$

from $t = 0$ to $t = 1$. [4]

(*b*) Evaluate :

$$\iint_S \overline{F} \cdot d\overline{s}$$

where

$$\overline{F} = 4xz\overline{i} + xyz^2\overline{j} + 3z\overline{k}$$

and S is the surface of the cone :

$$z^2 = x^2 + y^2$$

bounded by $z = 4$. [5]

(*c*) Apply Stokes' theorem to calculate :

$$\int_C 4y\,dx + 2z\,dy + 6y\,dz,$$

where C is the curve of intersection of

$$x^2 + y^2 + z^2 = 6z$$

and $z = x + 3$. [4]

Or

6. (*a*) Evaluate :

$$\int_C \overline{F} \cdot d\overline{r}$$

for

$$\overline{F} = x^2\overline{i} + 2xy\overline{j} + z\overline{k}$$

and C is the straight line joining (1, 0, 2), (3, 1, 1). [4]

(*b*) Show that : [4]

$$\iint_S \frac{\overline{r}}{r^3} \cdot \hat{n}\, ds = 0 .$$

(*c*) Evaluate using Stokes' theorem : [5]

$$\int_C (ydx + zdy + xdz)$$

C being intersection of

$$x^2 + y^2 + z^2 = a^2, \ x + z = a.$$

7. (*a*) A string is stretched and fastened to two points l apart. Motion is started by displacing the string into the form

$$y = k(lx - x^2)$$

from which it is released at time $t = 0$. Find the displacement of any point on the string at a distance of x from one end at time t. Using differential equation : [7]

$$\frac{\partial^2 y}{\partial t^2} = C^2 \frac{\partial^2 y}{\partial x^2}.$$

(*b*) The temperature at any point of the insulated metal rod of one metre length is governed by the differential equation :

$$\frac{\partial u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial t^2}.$$

Find $u(x, t)$ subject to the following :

(*i*) $u(0, t) = 0°C$

(*ii*) $u(1, t) = 0°C$

(*iii*) $u(x, 0) = 50°C$. [6]

Or

8. (*a*) Solve the partial differential equation :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

subject to boundary conditions :

(*i*) $u(0, y) = 0$

(*ii*) $u(x, 0) = 0$

(*iii*) $u(a, y) = 0$

(*iv*) $u(x, b) = 40$. [7]

(*b*) Use Fourier transform to solve the equation :

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \infty, \quad t > 0$$

subject to the following conditions :

(*i*) $u(0, t) = 0, \qquad t > 0$

(*ii*) $u(x, 0) = \begin{cases} 1 & 0 < x < 1 \\ 0 & x > 1 \end{cases}$

(*iii*) $u(x, t)$ is bounded. [6]