Seat No.	

[4956]-7

F.E. (Second Semester) EXAMINATION, 2016

ENGINEERING MATHEMATICS—II

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

***N.B.* :—** (*i*) Section I : Solve Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**.

Section II : Solve Q. No. **7** *or* Q. No. **8**, Q. No. **9** *or* Q. No. **10**, Q. No. **11** *or* Q. No. **12**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of non programmable calculator is allowed.

(*v*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Form the differential equaiton whose general solution is :

$$Y = Ax^2 + Bx^3,$$

where A and B are arbitrary constant. [6]

(*b*) Solve any *two* : [10]

(*i*) $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$

(*ii*) $\frac{dy}{dx} = -\frac{(y\cos x + \sin y + y)}{\sin x + x\cos y + x}$

(*iii*) $x^2(x^2-1)\frac{dy}{dx} + x(x^2+1)y = x^2 - 1$

Or

2. (*a*) Form the differential equation whose general solution is :

$$(x-h)^2 + (y-k)^2 = 1,$$

where h and k are arbitrary conotant. [6]

(*b*) Solve any *two* : [10]

(*i*) $3y^2 \dfrac{dy}{dx} + 2xy^3 = 4xe^{-x^2}$

(*ii*) $(x^2 + y^2 + x)\, dx + 2y\, dy = 0$

(*iii*) $x\dfrac{dy}{dx} + \dfrac{y^2}{x} = y$.

3. Solve any *three* : [18]

(*i*) A body originally at 80ºC cools down to 60ºC in 20 minutes the temperature of the air being 40ºC. What will be the temperature of the body after 40 minutes from the original ?

(*ii*) In a circuit containing inductance L, resistance R and voltage E1 the current I is given by $E = RI + L\dfrac{dI}{dt}$ Given L = 640 H, R = 200 W and E = 500 V. I being zero when t = 0, find the time that elapses before it reaches 90% of its maximum value.

(*iii*) The distance x descended by a parachuter satisfies differential equation :

$$\frac{dv}{dx} = 9\left(1 - \frac{v^2}{k^2}\right),$$

where v is the velocity, k and g are constant. If $v = 0$ and $x = 0$ at $t = 0$ show that :

$$x = \frac{k^2}{g} \log \cosh \left(\frac{gt}{k}\right)$$

(*iv*) Find the orthogonal trajecties of the family of curve $y^2 = 4ax$.

Or

4. Solve any *three* : [18]

(*i*) If 5% of radioactive substance disappeared in 50 years, how much will remain after 100 years.

(*ii*) The temp of air is 30ºC and the substance cools from 100ºC to 70°C in 15 minutes. Find the time when the temp will be 40°C.

(*iii*) An electric circuit contains an inductance of 5 henries and a resistance of 12 Ω in series with an emf 120 sin $20t$ volts. Find the current at $t = 0.01$, if it is zero when $t = 0$.

(*iv*) A particle is moving in a straight line with an acceleration

$$k\left[x + \frac{a^4}{x^3}\right]$$

directed towards origin, prove that it will arive at origin at the end of time $\frac{\pi}{4\sqrt{k}}$.

5. (*a*) Obtain the Fourier series for the period function $f(x) = x \sin x$ defined in the interval $0 \le x \le 2\pi$. [9]

(*b*) Establish the reduction formula connecting : [7]

$$I_n = \int_0^{\pi/2} x \cos^n x \, dx$$

with I_{n-2}, hence find I_4.

Or

6. (*a*) Obtain the constant term and the coefficient of the first cosine and sine terms in the expansion of y from the table : [8]

x	y
0	9
1	18
2	24
3	28
4	26
5	20

(*b*) Evaluate : [4]

$$\int_0^1 \frac{x dx}{\sqrt{\log\left(\frac{1}{x}\right)}}.$$

(*c*) Prove that : [4]

$$\int_0^\infty \frac{x^{m-1}}{(a+bx)^{m+n}} dx = \frac{1}{a^n b^m} B(m, n).$$

SECTION II

7. (*a*) Trace the following curves (any *two*) : [8]

(*i*) $y^2 = x^5 (2a - x)$

(*ii*) $r = a \cos 2\theta$

(*iii*) $x = at, \quad y = \frac{a}{t}$.

(*b*) Prove that : [4]

$$\int_0^1 \left(\frac{x^a - 1}{\log x} \right) dx = \log (1 + a)$$

$$a \geq 0$$

(*c*) Find the length of the arc of the curve $r = a\, e^{m\theta}$ intercepted between radii vectors r_1 and r_2. [5]

Or

8. (*a*) Trace the following curves (any *two*) : [8]

(*i*) $y^2 (x^2 - 1) = x$

(*ii*) $r = a \sin 3\theta$

(*iii*) $r = \left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$.

(*b*) Prove that : [4]

$$\int_0^\infty e^{-st} erf\left(\sqrt{t}\right) dt = \frac{1}{s\sqrt{s+1}}$$

(*c*) Find the length of the arc of the curve : [5]

$$x = e^\theta \left(\sin \frac{\theta}{2} + 2 \cos \frac{\theta}{2} \right), \qquad y = e^\theta \left(\cos \frac{\theta}{2} - 2 \sin \frac{\theta}{2} \right)$$

from $\theta = 0$ to $\theta = \pi$.

9. (*a*) Prove that the plane $x + y + z = 1$ touches the sphere :

$$3(x^2 + y^2 + z^2) - 30x + 12y - 18z + 89 = 0$$

and find the coordinates of the point of contact. [6]

(*b*) Find the equation of the right circular cone whose vertex is (1, 0, 1) which passes through the point (1, 1, 1) and axis of the cone is equally inclined with the coordinate axes. [5]

(*c*) Find the equation of the right circular cylinder with radius 2, whose axis passes through (1, 2, 3) and has direction cosines proportional to 2, –3, 6. [6]

Or

10. (*a*) Find the equation of the sphere which passes through the point (–1, 0, 0) and touches the plane $2x - y - 2z - 4 = 0$ at the point (1, 2, –2). [6]

(*b*) Find the equation of the right circular cone with vertex at origin, axis is the y-axis and semi-vertical angle is 30º. [5]

(*c*) Find the equation of the right circular cylinder with radius 2, whose axis is the line : [6]

$$\frac{x-1}{2} = \frac{y}{3} = \frac{z-3}{1}$$

11. Solve any *two* :

(*a*) Evaluate : [8]

$$\int_0^1 \int_0^{1-x} xt\,\sqrt{1-x-y}\; dydx$$

(*b*) Find the area bounded by : [8]

$$y^2 = 4ax \quad \text{and} \quad x^2 = 4ay.$$

(*c*) Find the C-G of the loop of the curve : [8]

$$r^2 = a^2 \cos 2\theta$$

Or

12. Solve any *two* :

(*a*) Evaluate : [8]

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{\cos^{-1} x \, dxdy}{\sqrt{(1-x^2-y^2)(1-x^2)}}$$

(*b*) Evaluate : [8]

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-y^2}} ydxdydz$$

(*c*) Find the M.I. of the area in XOY plane bounded by :

$$y^2 = 2x \text{ and } y = x,$$

assuming constant density ? [8]