

Total No. of Questions—8]

[Total No. of Printed Pages—4+2

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[4956]-101

F.E. (Common) EXAMINATION, 2016
ENGINEERING MATHEMATICS—I
(2012 PATTERN)

Time : Two Hours

Maximum Marks : 50

- N.B. :—** (i) Attempt Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4, Q. No. 5 or Q. No. 6, Q. No. 7 or Q. No. 8.
- (ii) Neat diagrams must be drawn wherever necessary.
- (iii) Figures to the right indicate full marks.
- (iv) Use of non-programmable electronic pocket calculator is allowed.
- (v) Assume suitable data, if necessary.

1. (a) Show that the system

$$3x + 4y + 5z = \alpha,$$

$$4x + 5y + 6z = \beta,$$

$$5x + 6y + 7z = \gamma$$

is consistent only when $2\beta = \alpha + \gamma$. [4]

- (b) Find eigen values and eigen vector corresponding to lowest eigen value for the matrix. [4]

$$A = \begin{bmatrix} 0 & 2 & 0 \\ 3 & -2 & 3 \\ 0 & 3 & 0 \end{bmatrix}$$

P.T.O.

(c) If [4]

$$\operatorname{cosec} \left(\frac{\pi}{4} + ix \right) = u + iv$$

where x, u, v are real, show that

$$(u^2 + v^2)^2 = 2(u^2 - v^2)$$

Or

2. (a) Examine for linear dependence of vectors [4]

(1, 2, -1, 0), (1, 3, 1, 2), (4, 2, 1, 0), (6, 1, 0, 1).

(b) Show that [4]

$$\left| \frac{z}{|z|} - 1 \right| \leq |\arg z|$$

(c) Prove that i^i is wholly real and find its principal value. [4]

3. (a) Test the convergence of the series (any one) [4]

$$(i) \quad 1 + \frac{3}{2!} + \frac{3^2}{3!} + \frac{3^3}{4!} + \frac{3^4}{5!} + \dots$$

$$(ii) \quad \sum \frac{1}{n} \cos \left(\frac{1}{n} \right)$$

(b) Expand

$$\sin^{-1} \left(\frac{2x}{1+x^2} \right)$$

in ascending powers of x [4]

- (c) Find the n^{th} derivative of [4]

$$\frac{1}{(x-1)^2(x-2)}.$$

Or

4. (a) Solve any one : [4]

(i) $\lim_{x \rightarrow 0} (1 + \tan x)^{\cot(x)}$

- (ii) Find the values of a and b such that

$$\lim_{x \rightarrow 0} \frac{x(1 + a \cos x) - b \sin x}{x^3} = 1.$$

- (b) Using Taylor's theorem expand [4]

$$x^4 - 3x^3 + 2x^2 - x + 1$$

in powers of $(x - 3)$

- (c) If

$$y = (\sin^{-1} x)^2,$$

prove that [4]

$$(1 - x^2)y_n + 2 - (2n + 1)x y_n + 1 - n^2 y_n = 0.$$

5. Solve any two questions :

- (a) If [6]

$$u = \log (x^3 + y^3 + z^3 - 3xyz),$$

prove that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = - \frac{9}{(x + y + z)^2}.$$

(b) If [7]

$$z = x^8 f\left(\frac{y}{x}\right) + y^{-8} \phi\left(\frac{x}{y}\right),$$

prove that

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 64z + 8y^{-8} Q\left(\frac{x}{y}\right) - 8x^8 f\left(\frac{y}{x}\right)$$

(c) If [6]

$$u = f(2x - 3y, 3y - 4z, 4z - 2x),$$

then find value of

$$\frac{1}{2} \frac{\partial u}{\partial x} + \frac{1}{3} \frac{\partial u}{\partial y} + \frac{1}{4} \frac{\partial u}{\partial z}.$$

Or

6. Solve any *two* questions :

(a) If [6]

$$ux + vy = 0, \quad \frac{u}{x} + \frac{v}{y} = 1$$

prove that

$$\left(\frac{\partial u}{\partial x}\right)_y - \left(\frac{\partial v}{\partial y}\right)_x = \frac{y^2 + x^2}{y^2 - x^2}.$$

(b) If [6]

$$u = \sec^{-1} \left[\frac{x + y}{\sqrt{x} + \sqrt{y}} \right],$$

prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\frac{1}{4} \cot u [3 + \cot^2 u].$$

(c) If

$$\phi = f(x, y, z), \quad x = \sqrt{vw}, \quad y = \sqrt{uw}, \quad z = \sqrt{uv},$$

prove that :

$$x \frac{\partial \phi}{\partial x} + y \frac{\partial \phi}{\partial y} + z \frac{\partial \phi}{\partial z} = u \frac{\partial \phi}{\partial u} + v \frac{\partial \phi}{\partial v} + w \frac{\partial \phi}{\partial w}. \quad [7]$$

7. (a) If

$$x = uv \quad \text{and} \quad y = \frac{u + v}{u - v},$$

$$\text{find } \frac{\partial(u, v)}{\partial(x, y)}. \quad [4]$$

(b) Find the percentage error in the area of an ellipse when an error of 1% each is made in measuring its semimajor and semiminor axes. [4]

(c) Find the extreme values of : [5]

$$f(x, y) = xy(a - x - y).$$

Or

8. (a) Examine whether the following functions are functionally dependent, if so find the relation between them [4]

$$u = \frac{x + y}{1 - xy}, \quad v = \tan^{-1} x + \tan^{-1} y.$$

- (b) A balloon is in the form of right circular cylinder of radius 1.5 m and length 4 m and is surrounded by hemispherical ends. If the radius is increased by 0.01 m and the length by 0.05 m, find the percentage change in the volume of a balloon. [4]
- (c) As the dimensions of a triangle ABC are varied, show that the maximum value of $\cos A \cos B \cos C$ is obtained when the triangle is equilateral. [5]