Total No. of Questions : 8]

SEAT No. :

P3867

[Total No. of Pages : 3

[4960] - 1064

M.E. (Mechanical) (Design Engineering)

ADVANCED MATHEMATICS

(2013 Pattern) (Semester - I)

Time : 3 Hours] *[Max. Marks :50*

Instructions to the candidates:

1) ***Answer any five questions.***
2) ***Neat diagrams must be drawn wherever necessary.***
3) ***Figures to the right indicate full marks.***
4) ***Use of non programmable electronic pocket calculator is allowed.***
5) ***Assume suitable data, if necessary.***

Q1) a) Find an orthonormal basis for the Euclidean space R^3, by applying Gram-Schmidf's method to the following vectors (1, –1, 0), (0, 1, 0) and (2, 3, 1). **[5]**

b) An electrostatic field in the xy plane is given by the potential function $\phi = x^3 - 3xy^2 + 2x$, find the stream function. **[5]**

Q2) a) Evaluate $\oint \frac{12z-7}{(z-1)^2(2z+3)}dz,$ where c is the circle $|z| = 2$ **[5]**

b) Find the Laplace transform of the given function. **[5]**

i) $(t + 2)\, u\,(t - 3) + e^{2t} \sin 3t\, \delta\,(t - 2)$.

ii) $t \operatorname{erf} \sqrt{t}$.

Q3) a) Find the mechanical system which is governed by the differential equation and with given intial conditions by Laplace transform **[5]**

$$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 1 - e^{2t}$$

with $x(0) = 1$ and $x'(0) = 0$

b) Solve by series method the following differential equation

$$\frac{d^2y}{dx^2} + x^2y = 0$$ **[5]**

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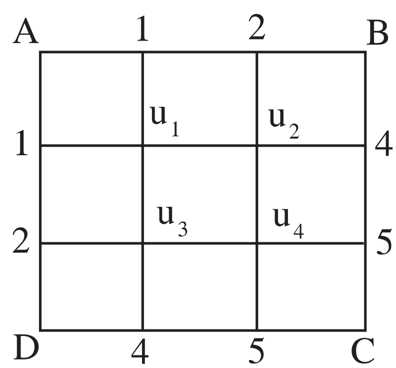
Q4) a) Find the largest eigen value and corresponding eigen vector of the matrix by power method where **[5]**

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \text{taking } X_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

b) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ for the given square mesh with boundary values as shown in the following figure. **[5]**



Q5) a) Given $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$ with boundary conditions $u(0, t) = 0 = u(8, t)$ and $u(x, 0) = 4x - \frac{1}{2}x^2$ at the points $x = i$; $i = 0, 1, 2, ...8$. Find the values of $u(x, t)$ upto 5 levels. **[5]**

b) Using Galerkin's method, obtain an approximate solution of the boundary value problem **[5]**

$$\frac{d}{dx}\left(x\frac{dy}{dx}\right) + y = x, \quad y(0) = 0, \quad y(1) = 1$$

Q6) a) Find the curves on which the functional $\int_1^2 \left[x^2(y')^2 + 2y(x+y)\right]dx$ with $y(1) = y(2) = 0$ can be extremized. **[5]**

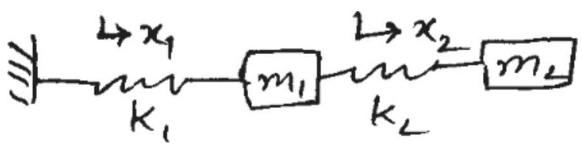
b) Solve the system of equations by least square method $3x - 2y = 1$, $x + y = -1$, $x + 2y = 3$. **[5]**

Q7) a) Show that the transformation $w = \dfrac{i(1-z)}{1+z}$ maps the circle $|z| = 1$ into the real axis of the w plane and interior of the circle $|z| < 1$ into the upper half of the w-plane. **[5]**

b) Find the Fourier cosine transform of the function **[5]**

$$f(x) = \begin{cases} x & for \quad 0 < x < 1 \\ 2 - x & for \quad 1 < x < 2 \\ 0 & for \quad x > 2 \end{cases}$$

Q8) a) For the system of masses & spring in the figure below $m_1 = 2$, $m_2 = 1$, $k_1 = 4$ and $k_2 = 2$, assuming there is no friction. Find natural frequencies of the system and corresponding normal modes of vibration using matrix method. **[5]**



b) Solve the boundary value problem $U_{tt} = U_{xx}$ upto $t = 0.5$ with a spacing 0.1. Subject to $u(0, t) = 0 = u(1, t)$, $u_t(x, 0) = 0$ $u(x, 0) = 10 + x(10 - x)$. **[5]**