Total No. of Questions : 8]

SEAT No. :

P3886

[Total No. of Pages : 3

[4960] - 1100

M.E. (Mechanical) (Design Engineering)

ADVANCED MATHEMATICS

(Design/CAD-CAM/Auto Mobile)

(2013 Pattern)

Time : 3 Hours] *[Max. Marks :50*

Instructions to the candidates:

1) ***Answer any five questions.***
2) ***Neat diagrams must be drawn wherever necessary.***
3) ***Figures to the right indicate full marks.***
4) ***Use of non-programmable electronic pocket calculators is allowed.***
5) ***Assume suitable data, if necessary.***

Q1) a) Find an orthonormal basis for the Euclidean space R^3 by applying Gram-Schmidt's method to the following vectors; (1, –1, 0), (0, 1, 0) and (2, 3, 1). **[5]**

b) The complex potential for an electrostatic field is given $\omega = \phi + i\chi$, which is analytic. If $\phi + \chi = e^{-x}(\cos y - \sin y)$, find the function $\omega = f(z)$ **[5]**

Q2) a) Evaluate $\oint \frac{z-2}{z^2+1}\,dz$, where c is $|z - i| = ½$ **[5]**

b) Solve the differential equation of a system given as $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = e^{-2t}$ with $y(0) = 0$, $y'(0) = 0$, by using Laplace transform. **[5]**

Q3) a) Find the Laplace transform of half wave rectified sine wave defined as

$$f(t)=\begin{cases}\sin(\omega t), & 0<t<\pi/\omega \\ 0, & \pi/\omega<t<\dfrac{2\pi}{\omega}\end{cases}$$ **[5]**

and $f\left(t+\dfrac{2\pi}{\omega}\right)=f(t)$

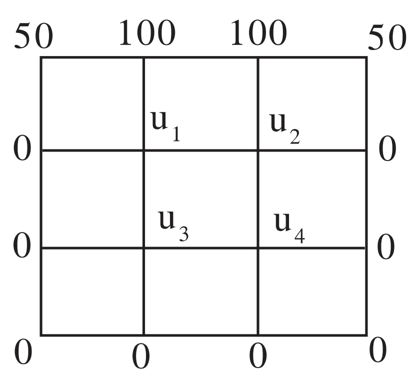
b) Solve the following differential equation by series solution method

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0.$$ **[5]**

Q4) a) Find the dominant eigen value and the corresponding eigen vector by power method of **[5]**

$$A=\begin{pmatrix}1 & 6 & 1\\ 1 & 2 & 0\\ 0 & 0 & 3\end{pmatrix} \text{taking } X_0=\begin{pmatrix}1\\0\\0\end{pmatrix}$$

b) Solve the boundary value problem $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}=0$ for the given square of sides three units. **[5]**



Q5) a) Solve the equation $2u_{xx}-u_t=0$, $u(x,0)=50(4-x)$, $u(0,t)=0$, $u(4,0)=0$, with $h=1$, upto $t=1$. **[5]**

b) Using Galerkin's method, solve the boundary value problem **[5]**

$$\frac{d}{dx}\left(x\frac{dy}{dx}\right)+y=x,\ y(0)=0,\ y(1)=1$$

Q6) a) Find the extremal's of the functional and extremise value of $\int_0^2 (x - y')^2 \, dx$ subject to $y(0) = 0$, $y(2) = 4$. **[5]**

b) Solve the linear equation system using least square method $x - y = 1$, $x + 2y = 3$, & $x - 2y = -1$. **[5]**

Q7) a) Show that the function $\omega = \frac{4}{z}$ transforms the straight line $x = c$ in the zplane into a circle in the w-plane. **[5]**

b) Find the Fourier cosine transform of **[5]**

$$f(x) = \begin{cases} x & for \quad 0 < x < ½ \\ 1 - x & for \quad ½ < x < 1 \\ 0 & for \quad x > 1 \end{cases}$$

Q8) a) The system shown in the figure begins to vibrate with initial displacement $y_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ assuming that there is no friction determine the subsequent motion $[k_1 = 1, k_2 = 1, k_3 = 1, m_1 = m_2 = 1]$ **[5]**

b) Solve the equation $U_{tt} = U_{xx}$, $u(x, 0) = \frac{x(100 - x)}{100}$, $u(0, t) = 0$, $u(10, t) = 0$, $u_t(x, 0) = 0$ for $x = 0(1) = 10$; $t = 0(1)5$. **[5]**