Total No. of Questions—**12**]

[Total No. of Printed Pages—**7**

Seat No.	

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S.E. (Civil) (First Semester) EXAMINATION, 2016

ENGINEERING MATHEMATICS—III

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

N.B. **:—** (*i*) Answer *three* questions from Section I and *three* questions from Section II.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Figures to the right indicate full marks.

(*v*) Use of non-programmable electronic pocket calculator is allowed.

(*vi*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Solve the following (any *three*) : [12]

(*i*) $(D-1)^3 y = e^x + 2^x + 3$

(*ii*) $(D^2 - 4D + 3)y = x^3 e^{2x}$

(*iii*) $(D^2 - 2D + 1)y = x e^x \sin x$

(*iv*) $(D^2 + 1)y = \cot x$ – (by variation of parameters)

(*v*) $x^2 \frac{d^2y}{dx^2} + 5x \frac{dy}{dx} + 3y = \frac{\log x}{x^2}$.

(*b*) Solve : [5]

$$\frac{dx}{3z-4y} = \frac{dy}{4x-2z} = \frac{dz}{2y-3x}.$$

P.T.O.

Or

2. (*a*) Solve the following (any *three*) : [12]

(*i*) $(D^2 + 3D + 2)y = e^{e^x} - \sin(e^x)$

(*ii*) $(D^2 + 5D + 4)y = x^2 + 7x + 9$

(*iii*) $(D^2 - 3D + 2)y = x^2 + \sin x$

(*iv*) $(D^2 - 3D^2 + 3D - 1)y = \sqrt{x}\, e^x$

(*v*) $(2x + 1)^2 \cdot \dfrac{d^2y}{dx^2} + 2(2x + 1)\dfrac{dy}{dx} + 4y = 4 \sin [\log (2x + 1)^2].$

(*b*) Solve : [5]

$$\frac{du}{dx} + V = \sin x; \qquad \frac{dv}{dx} + u = \cos x$$

Given :

$$u = 1; v = 0 \text{ at } x = 0 \ldots\ldots$$

3. (*a*) A horizontal beam is uniformly loaded, its one end fixed and other end is subjected to a tensile force P. The deflection of beam is given by $EI \dfrac{d^2y}{dx^2} = Py - \dfrac{Wx^2}{2}$.

Given that :

$$\frac{dy}{dx} = 0 \text{ at } t = 0.$$

Show that the deflection of the beam for the given x is :

$$y = \frac{w}{2Pn^2}[2(1 - \cosh nx) + n^2x^2];$$

where :

$$n^2 = \frac{P}{EI}.$$ [8]

(b) Find the deflection $y(x, t)$ of the vibrating string of length π and ends are fixed, corresponding to zero initial velocity. Initially the velocity is zero and the deflection : [8]

$$f(x) = k[\sin x - \sin 2x] \quad \text{at} \quad t = 0,$$

given $C^2 = 1$.

Or

4. (a) A body of mass 20 kg is attached to one end of an elastic spring of which other end is fixed. A damping force is acting on mass is 200 times the velocity at the instant. If weight 10 N produces elongation of 2 cm in the spring, find the differential equation of motion of the spring. If $x = 0$ and $v = 30$ m/s at $t = 0$, determine the position of body. [8]

(b) Solve : [8]

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}.$$

if :

(i) u is finite for all t

(ii) $u = 0$ at $x = 0$

(iii) $u = 0$ at $x = L$

(iv) $u(x, 0) = u_0$, $0 \le x \le L$; where L is length of bar.

5. (a) Solve the following system of equations by Gauss elimination method : [9]

$10x + 2y + z = 9$, $2x + 20y - 2z = -44$; $-2x + 3y + 10z = 22$.

(b) Use Euler's modified method, find : [8]

$$y(1.2), \text{ given } \frac{dy}{dx} = \frac{1}{x^2} - \frac{y}{x}; \quad y(1) = 1$$

with $h = 0.1$.

Or

6. (*a*) Solve by Gauss-Seidal iteration method : [8]

$$20x + y - 2z = 17, \; 3x + 20y - z = -18;$$

$$2x - 3y + 20z = 25.$$

(*b*) Apply fourth order Runge-Kutta method to solve : [9]

$$\frac{dy}{dx} = 3x + \frac{y}{2}; \; y(0) = 1$$

Determine $y(0.2)$, taking $h = 0.1$.

SECTION II

7. (*a*) Find the first four moments about the working mean 44.5 of a distribution are –0.4, 2.99, –0.08 and 27.63. Calculate the moments about the mean. Also calculate β_1 and β_2. [6]

(*b*) Calculate the coefficient of correlation for the following data : [6]

x_i	y_i
35	32
34	30
40	31
43	32
56	53
20	20
38	33

(*c*) A can hit the target 1 out of 4 times.

B can hit the target 2 out of 3 times.

C can hit the target 3 out of 4 times.

Find the probability that at least two hits the target. [5]

Or

8. (*a*) Two lines of regression are : [7]

$$5y - 8x + 17 = 0;\ 2y - 5x + 14 = 0.$$

If

$$\sigma_y^2 = 16.$$

Find :

(*i*) The mean value of x and y.

(*ii*) σ_x^2

(*iii*) $r(x, y)$.

(*b*) 20% of bolts produced by machine are defective. Determine the probability that out of 4 bolts chosen at random : [5]

(*i*) one is defective.

(*ii*) at most two bolts are defective.

(*c*) In a male population of 1000, the mean height is 8.16 inches and S.D. is 3.2 inches. How many men will be expected to be more than 6 feet (Area under $z = 1.2$ is 0.3849). [5]

9. (*a*) Find the angle between two surfaces : [5]

$$z = x^2 + y^2 \text{ and } xy^3z^2 = 4 \text{ at } (1, 2, 5) \text{ ....}$$

(*b*) Find the directional derivative of [6]

$$\phi = x^2 - y^2 + 2z^2 \text{ at the point } P(1, 2, 3) \text{ ......}$$

in the direction of line PQ, where Q is (5, 0, 4).

(*c*) Prove that : [5]

$$\bar{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$$

is irrotational force field. Also find the corresponding scalar potential.

Or

10. (*a*) Prove that (any *two*) : [6]

(*i*) $\nabla^4 (r^2 \log r) = \frac{6}{r^2}$

(*ii*) $\nabla^2 \left(\frac{\bar{a} . \bar{b}}{r} \right) = \bar{0}$.

(*iii*) For a scalar function ϕ and ψ :

$$\nabla . (\phi \nabla \psi - \psi \nabla \phi) = \phi \nabla^2 \psi - \psi \nabla^2 \phi.$$

(*b*) Find constants a and b so that the surface $ax^2 - byz = (a + 2)x$ will be orthogonal to the surface $4x^2y = 4 - z^3$ at the point $(1, -1, 2)$. [5]

(*c*) If the directional derivative of $\phi = axy + byz + czx$ at $(1, 1, 1)$ has maximum magnitude 8 in the direction parallel to y-axis. Find the values of a, b, c. [5]

11. (*a*) Find the work done by $\bar{F} = x^2 \bar{i} + yz \bar{j} + z \bar{k}$ in moving a particle along the straight line segment from $(1, 2, 2)$ to $(3, 4, 4)$. [5]

(*b*) Evaluate : [6]

$$\iint_S \bar{F} . \hat{n} \, dS,$$

where :

$$\bar{F} = x^3 \bar{i} + y^3 \bar{j} + z^3 \bar{k}$$

and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ (Use divergence theorem).

(*c*) Evaluate : [6]

$$\iint_S (\nabla \times \bar{F}) . d\bar{S},$$

where :

$$\bar{F} = (x^2 - yz) \bar{i} - (x - y) \bar{j} + 3x^2 y^2 \bar{k}$$

and S is the curved surface of cone

$$z^2 = x^2 + y^2$$

bounded by $z = 4$.

Or

12. (*a*) Evaluate [5]

$$\oint_C (\sin y - y^3)\, dx + (xy^2 + x \cos y) dy$$

by Green's theorem, $C : x^2 + y^2 = a^2$.

(*b*) Using Stokes theorem evaluate : [6]

$$\int_C \overline{F} . d\overline{r},$$

where

$$\overline{F} = y^2 \overline{i} + x^2 \overline{j} - (x + z)\overline{k}$$

and C is the boundary of triangle with vertices (0, 0, 0), (1, 0, 0) and (1, 1, 0).

(*c*) If the velocity potential of a fluid motion is given by $\phi = \log(xyz)$, find the equation of streamlines. [6]